

Part-Time Employment and Firm-Level Labor Demand over the Business Cycle

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Abstract

Part-time employment for economic reasons (*PTE*) is countercyclical, volatile, and transitory. Workers in *PTE* are nearly three times more likely than the unemployed to return to full-time work in a given month, and seven times more likely than full-time workers to become unemployed. Using household survey data, I demonstrate that cyclical fluctuations in *PTE* come from changes in the transition rates between full-time and part-time employment rather than between part-time and unemployment. Moreover, these movements are primarily due to within-job changes in hours. Accordingly, I model part-time work focusing on a firm's decision to hire, fire, or partially utilize its labor force. Firms in the model are heterogeneous in size and productivity, and are subject to search frictions. The model produces firm-level utilization of part-time employment which is consistent with observed worker flows, and varies across the size and age distributions of firms. Over the business cycle, the model matches the observed relative volatility of unemployment and *PTE*. Part-time labor utilization by firms increases the volatility of vacancies and unemployment in the model relative to the case with only an extensive margin.

1 Introduction

Part-time employment for economic reasons (*PTE*) has recently attracted attention from policy-makers as an indicator of weakness in the aggregate labor market due to its unprecedented size during the last recession. *PTE* reached 7% of employment at its peak, affecting

*Preliminary. Any opinions and conclusions expressed herein are those of the author and do not necessarily represent the views of the U.S. Census Bureau

roughly 9 million workers in the US. Like the unemployment rate, the share of employment in *PTE* is volatile and countercyclical. Our understanding of part-time labor and its relation to the aggregate economy, however, is lacking relative to our knowledge of unemployment. This paper focuses on examining the changes in the population of *PTE* over the business cycle and developing a search model of involuntary part-time work.

I begin by using data from the Current Population Survey to document three important facts about *PTE*: first, more full-time workers become part-time employed for economic reasons each month than become unemployed, and this inflow is more volatile than is the flow of full-time workers to unemployment. During the last recession and subsequent recovery, the decrease in aggregate hours caused by individuals moving from full-time to *PTE* is on average 77% of the loss in hours caused by full-time workers separating to unemployment. Second, *PTE* is not a persistent state, and is characterized by high flow probabilities to full-time employment and unemployment. Workers in *PTE* are three times as likely as the unemployed to return to full-time work, and about seven times as likely as full-time workers to separate to unemployment. Lastly, *PTE* fluctuates due to within-job changes in hours rather than to changes in unemployment status.¹ Using the dependent interview structure of the CPS as in Fallick and Fleischman (2004) to look at job transitions, I find that on average, 82% of the transitions of employed workers into or out of *PTE* are due to within-job changes in hours.

I build upon the framework of Kaas and Kircher (2015) on the basis of these facts to model firm-level labor demand with a part-time employment margin in a frictional labor market. This focus on firm-level demand is appropriate because the movement of workers with respect to *PTE* is primarily within-job. Firms are heterogeneous in productivity,

¹This result is also found in work by Canon et al. (2014), who use a methodology similar to Shimer (2012) to show that within-employment flows contribute more to the rise in *PTE* in the last recession than the flow of workers to or from unemployment. In addition to this result, they document the differences in wages, occupations, and demographic characteristics of workers in full-time and part-time work by reason and link these properties of workers in *PTE* to the job-polarization literature. I use the dependent interview structure of the CPS to show that the majority of the fluctuations in *PTE* are due to within-job changes rather than from job-to-job transitions.

and firm size is determined through a decreasing returns to scale production technology in labor. Firms expand and contract their respective workforces in response to idiosyncratic and aggregate shocks, while facing frictions due to costly vacancy postings and competitive search. Firms can use part-time labor when facing a negative productivity shock in order to lower wage costs temporarily and reduce layoffs, avoiding the costly recruitment necessitated by future hiring when productivity increases.

Given that the facts outlined in CPS data concern worker flows and the model is about firms, I focus on the ability of the model to reproduce the qualitative features of *PTE* observed in the CPS data. Part-time utilization within firms is volatile in the model, implying that the flows of workers between full-time and part-time employment are large relative to the movement of workers between full-time employment and unemployment. This matches the first fact I outlined previously: workers in full-time employment are more likely to move to part-time than to separate to unemployment. The volatility also reflects the finding that workers in part-time employment have a high probability of returning to full-time work. Firms use part-time labor in conjunction with separation, so that workers placed in *PTE* also experience higher separation rates than do full-time workers. The high respective probabilities of either returning to full-time or separating to unemployment are consistent with the lack of persistence in *PTE* indicated by the CPS data. Just as in the data, the aggregate stock of *PTE* is as volatile as unemployment and negatively correlated with output over the business cycle.

In quantitative exercises, I investigate the distribution of part-time usage across firms of varying growth rates and age, and within different size categories. I find that part-time utilization is correlated with employment growth for shrinking firms, reflecting firms' usage of *PTE* along with layoffs in response to negative shocks. The model is consistent with the positive cross-sectional correlations of employment and hours growth outlined in empirical work by Cooper et al. (2004, 2007) and Trapeznikova (2014) at the aggregate level. Consistent with their empirical findings on firm-level correlations of hours and employment

growth, firm-level usage of part-time leads employment growth in my model. To extend the analysis of part-time usage to observable firm characteristics, I calibrate the model to match the firm size and age distribution found in Business Dynamics Statistics data from the Bureau of Labor Statistics. Heterogeneity in a permanent component of firm-level productivity matches the firm size distribution in the data, while the entrant share of firm productivity types and firm exit probabilities match the age distribution of firms. Very young firms use less part-time employment due to high employment growth, and part-time usage increases with firm age. Heterogeneity in the exit rates of firms plays a key role in the distribution of part-time utilization. If exit rates were equal for all firms, part-time use would decrease with permanent productivity (and, therefore, size). Lower exit probabilities for high-productivity firms increase the value of future labor in the same manner as does a lower discount rate, incentivizing productive (and large) firms to use part-time employment instead of layoffs. Part-time utilization increases by about 40% for the largest firms once exit rates are allowed to vary.

Over the business cycle, the model generates countercyclical and volatile movements in both part-time employment and unemployment. The incorporation of a part-time employment margin increases the volatility of both unemployment and vacancies relative to the case with only an employment margin. Part-time employment affects the cyclical properties of the model through two mechanisms. First, part-time utilization increases the volatility of output relative to average labor productivity. Aggregate shocks cause a reduction in output from firms' immediate use of part-time employment. Due to decreasing returns to scale in labor, this decrease in labor utilization is accompanied by an increase in productivity per hour of labor. This effect is similar to that caused by time-varying effort in the labor hoarding and factor-utilization literature, as exemplified by Burnside et al. (1993), Burnside and Eichenbaum (1996), and Bils and Cho (1994). Burnside et al. (1993) assume fixed hours per worker prior to the realization of productivity shocks in addition to fixed employment. Firms vary labor-utilization by demanding varying levels of effort, producing volatility in

output and procyclical labor productivity. My model achieves similar effects by allowing for the adjustment of hours through the use of part-time employment by firms.

The second effect of the part-time margin in the model is an increase in the volatility of unemployment and vacancies from changes in the use of layoffs by firms. Firms use fewer separations in reaction to negative shocks when there is an operative part-time margin. Because idiosyncratic shocks generate the majority of endogenous worker separations, a model with part-time labor requires a less persistent shock process to generate the same unemployment rate as does a full-time only model. This increase in the prevalence of firm-level shocks produces more employment adjustment, increasing the volatility of unemployment and vacancies over the business cycle. The increased volatility in unemployment and vacancies is not accompanied by an increase in the volatility of the separation rate because layoffs are decreased in favor of part-time employment. The amplification of unemployment and vacancies over the business cycle in a model with large firms is also found in Elsby and Michaels (2013), but through a different mechanism. In their model, decreasing returns and the Stole-Zwiebel bargaining process for wages implies that workers and firms split the marginal and infra-marginal surplus of a match, resulting in a low surplus for the marginal worker, akin to the recalibration of the standard search model in Hagedorn and Manovskii (2008).

Section 2 describes and reports the facts I outline in CPS data. Section 3 presents a search model which is consistent with the documented data on part-time work. In section 4, I characterize the dynamics of the firm through its policy functions on hiring, part-time utilization, and layoffs. In Section 5, I calibrate the model to match aspects of the aggregate labor market as well as the size distribution of firms. Section 6 discusses the implications of the calibrated model for part-time use over the firm size and age distribution, as well as the business cycle properties of the model. Section 7 outlines ongoing work using US Census Bureau data to document the co-movements of hours and employment growth over the distribution of firms and concludes.

2 Part-time Employment in Household Survey Data

In this section, I use data on part-time employment in the Current Population Survey (CPS) to highlight several facts about part-time work.

1. Part-time for economic reasons is volatile and countercyclical, and co-moves with unemployment.
2. Part-time for economic reasons is not persistent, and is characterized by high flow probabilities back to full-time employment and to unemployment.
3. Fluctuations in part-time employment arise largely from within-employment (to and from FT or PTN) instead of between PTE and non-employment (unemployment, U or nonparticipation, N). Further, part-time employment transitions primarily occur within job rather than from job-to-job transitions.

These facts are important for understanding the mechanism through which cyclical movements in PTE arise. Particularly, I make the case that these fluctuations in part-time employment for economic reasons are driven primarily by firm-level demand or “slack work and business conditions” as opposed to coming from workers’ search and job-finding behavior (“failure to find a full-time job”).

2.1 Definitions

The Current Population Survey gives detailed information at the individual level on labor market participation, either in employment or nonemployment, and the weekly hours worked by detailed reason. The definition of full-time and part-time employed by reason given by the Bureau of Labor Statistics (BLS) follows from questions asked of employed workers about working hours and availability. The BLS definition and terminology is “At work” by full- or part-time and reason. A worker is referred to in this paper as “Full-time” if they are classified

by the BLS as “At work 35 or more hours.”² Workers who report working a total of 1-34 hours at all jobs during the reference week are classified as “At work part-time.” Part-time workers are also further classified by reason for working part-time. Workers who are classified as “At work part-time for economic reasons” (*PTE*) are individuals working part-time hours who report wanting and being available for full-time work and provide the reason for being at work part-time as either “Slack Work/Business Conditions,” “Failure to Find Full-time Work,” or “Seasonal Work/Changes in Demand.” Those who report being “At work part-time for non-economic reasons” (*PTN*) are individuals “At work part-time” who report reasons for working part-time that are not related to employer demand, or report that they are unwilling or unable to work full-time. In this paper, I refer to the BLS classifications of “At work full/part-time/part-time by reason” simply as “Part-time employed (for econ/non-econ. reasons)” and “Full-time employed.”³

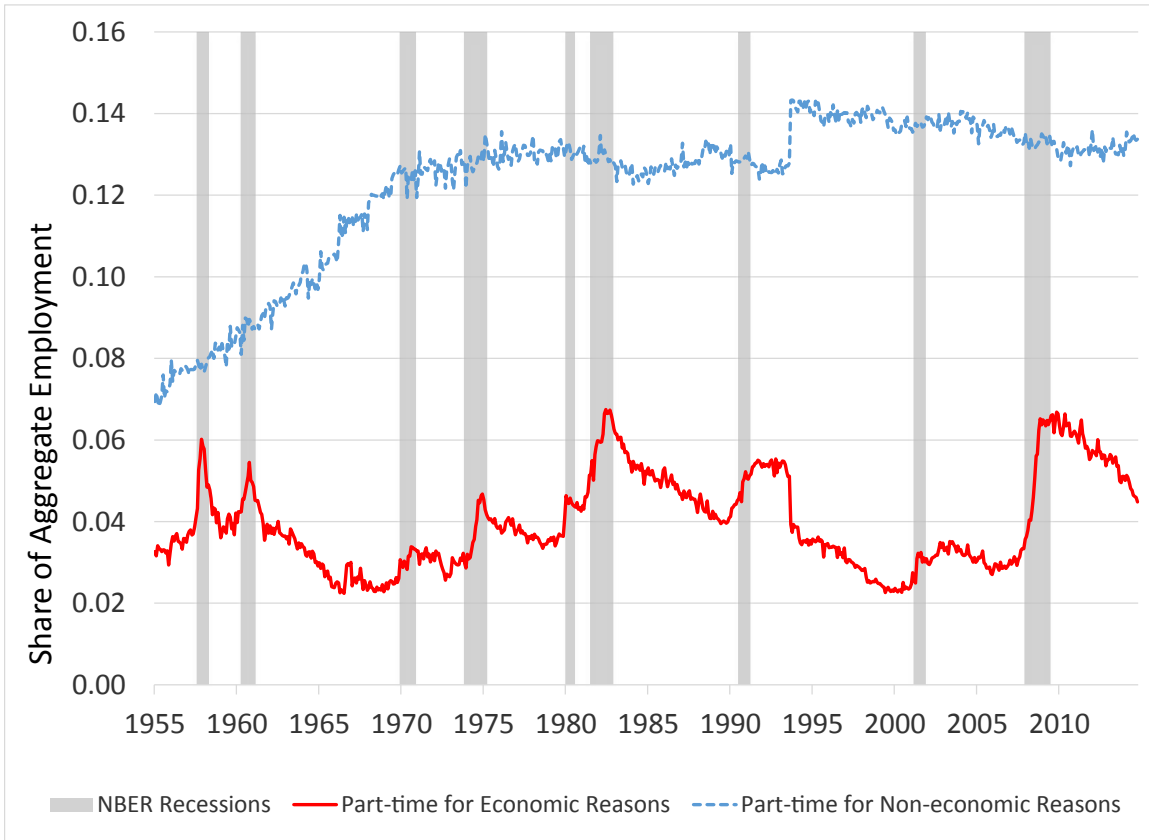
2.2 PTE is Countercyclical and Volatile

The volatility and cyclical nature of part-time employment by reason is easily illustrated by considering each group’s share of total employment. Figure 1 shows that part-time employed for non-economic reasons displays a secular increase until about 1970, after which it makes

²This is different from the typical definition of “employment” in that absent workers are not counted in this population. This definition relies only on actual hours worked, and does not necessarily coincide with being in the “usual” full-time labor force, which consists of those in the labor force who report usually working 35 or more hours per week (regardless of current employment or actual hours worked in the survey week). Similarly, the part-time labor force consists of employed and unemployed individuals who usually work less than 35 hours per week.

³The sample time-frame considered for the gross flows analysis of Part-time employment is 1994-2014, after the CPS redesign. The sample time-frame initially studied for the gross flows analysis was chosen due to the fact that there are substantial changes in the 1994 redesign affecting the classification of workers into full-time and part-time employed, especially by reason. Polivka and Miller (1998) document that prior to the redesign, the CPS survey structure only asked those who reported working less than 35 hours what was their usual hours worked per week. This leads to an underestimation of the size of the part-time labor force by 9.8% relative to the redesign. The 1994 redesign also corrected an over-estimate of the part-time employed for economic reasons. The reported number of of part-time employed for economic reasons is over-stated by 20% relative to the unrevised CPS. This difference is primarily due to the unrevised survey assuming desire for and availability to work full-time based on the reported reason given for being part-time employed, rather than asking this question directly. For details on the CPS variables and responses used to classify workers, see Appendix C.1

Figure 1: Part-time Employment’s Share of Aggregate Employment by Reason



CPS, May 1955 - December 2014, from the Federal Reserve Economic Database of the Federal Reserve Bank of St. Louis. Gray bars in figures indicate NBER recession dates.

up a roughly constant share of 14% of total employment.⁴ Although this group fluctuates cyclically, it does so in tandem with total employment. By contrast, part-time employment for economic reasons is markedly counter-cyclical, increasing to nearly 7% of employment after the 1980’s “double-dip” recession and again peaking at about 7% of employment in 2009. That is, part-time employment for economic reasons fluctuates significantly more than the stock of total employment over the business cycle.

To analyze the cyclical movements and volatility of the stocks of full and part-time labor, I compare them to the cyclical properties of quarterly aggregate output.⁵ The business cycle

⁴Except for the discrete jump due to the 1994 CPS redesign.

⁵I take the average of the monthly flows during the three months of each quarter. I then log and de-trend the quarterly series using a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 10^5$ as in Shimer (2005), and compare the standard deviations of the cyclical components of each variable along with their correlation with the cyclical component of logged and detrended aggregate output (Y).

Table 1: Cyclical Properties of Labor Market Stocks

	<i>Y</i>	<i>FT</i>	<i>PTN</i>	<i>PTE</i>	<i>u rate</i>
<i>std(x)</i>	.024	.027	.024	.174	.190
<i>std(x)/std(Y)</i>	1	1.15	1.00	7.39	8.08
<i>corrcoef(Y, x)</i>	1	.906	.826	-.861	-.795

Cyclical components of quarterly and quarterly averaged series, logged and HP Filtered with $\lambda = 100,000$

statistics of the stocks of full-time and part-time labor outlined in Table 1 confirm the cyclical patterns shown in Figure 1. Full-time labor and part-time employment for non-economic reasons have similar volatilities and are both pro-cyclical. The unemployment rate is counter-cyclical and much more volatile than output or employment. Part-time employment for economic reasons is nearly as volatile as the unemployment rate and also negatively correlated with aggregate output. Given that these statistics often motivate the interest in understanding the cyclical dynamics of the unemployment rate, it seems reasonable to devote attention to the cyclical dynamics of involuntary part-time labor.

Another measure of interest is the extent to which the transition of workers in and out of part-time for economic reasons accounts for the fluctuations in hours worked over the business cycle. While the total contribution to fluctuations in total hours from flows in and out of *PTE* is small, the changes in hours attributed to individual flows related to *PTE* can be large and cyclical. For instance, the total of net hours lost in a given month due to workers who moved from *FT* to *PTE* is nearly as large as the total hours lost by full-time workers who moved to unemployment. These patterns are explored further in ??.

2.3 PTE is Not Persistent

The average monthly transition probabilities between employment states in the CPS show the lack of persistence in *PTE* relative to other employment states. in Table 2, I show the average of the seasonally adjusted monthly transition probabilities of workers between states of full-time employed, at work part-time for non-economic reasons, at work part-time for

economic reasons, unemployment, and nonparticipation for the sample period 1994-2014.⁶

Table 2: US Average Monthly Flow Probabilities

From	To				
	FT	PTN	PTE	U	N
FT	.880	.080	.015	.009	.014
PTN	.287	.582	.043	.019	.066
PTE	.301	.225	.361	.064	.046
U	.120	.071	.047	.522	.240
N	.015	.022	.003	.026	.934

From Current Population Survey, 1994:1-2014:12

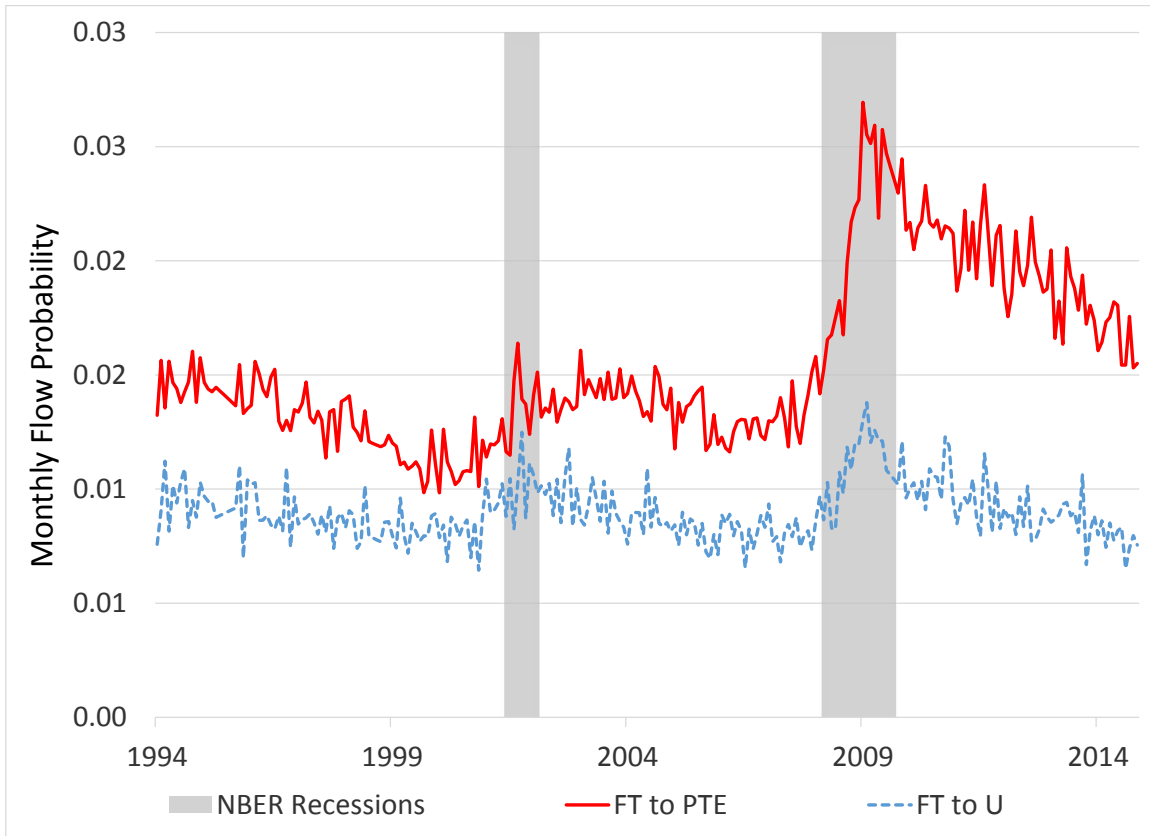
Dis-aggregating employment into full-time and part-time categories highlights some key differences in the flow probabilities for employed workers. Within-employment transitions are large, especially for *PTE* workers. The transition probability of full-time workers to part-time for economic reasons ($FT \rightarrow PTE$) is *larger* than the flow of full-time workers to unemployment ($FT \rightarrow U$). As seen in Table 2 or Figure 2, while about 1% of full-time workers become unemployed in a given month, 1.5% of workers transition to part-time for economic reasons. Although these transition probabilities seem small, the number of workers who move to both *PTE* and *U* from full-time work is large due to the size of the full-time employed population. More full-time workers move to *PTE* than to *U*.

Part-time for economic reasons is a very transitory state: only 36% of workers at work part-time for economic reasons remain in *PTE* the following month. By contrast, about half of unemployed workers remain unemployed in a given month. The flow of workers back to full-time employment is also much higher for the part-time employed than it is for the unemployed. While only 12% of unemployed workers move to a full-time job each month, nearly one-third of workers classified as *PTE* return to full-time employment. While part of the transitory nature of *PTE* is due to the high probability of returning to full-time work, workers in *PTE* also have a high probability of moving to *PTN* and nonemployment. 6.4%

⁶The CPS flow data is constructed from a modified version of code written by Robert Shimer. For additional details, please see Shimer (2007) and his webpage. <http://sites.google.com/site/robertshimer/research/flows>

of workers in *PTE* become unemployed on average each month, a rate that is 7 times higher than for full-time workers.

Figure 2: Monthly Probability of Exiting Full-time Employment



The dashed blue line plots the monthly transition probability of full-time workers to unemployment. The solid red line plots the monthly transition probability of full-time workers to part-time for economic reasons. The flow probabilities are constructed from CPS micro data from 1994-2014. Gray bars indicate NBER recession dates.

2.4 PTE Status Fluctuates due to Within-job Movements

Here, I show that the fluctuation in the *PTE* stock comes from changes in the flow-probabilities of employed workers to and from *PTE* rather than changes in the flows to or from non-employment. Looking at the reasons reported in the CPS for why a worker may be at work part-time highlights the two different ways in which workers may become underemployed. Workers can be classified as *PTE* due to slack work or business conditions, or they may be unable to find a full-time job. The first reason would indicate that firm-level

demand for currently employed workers drives changes in PTE , and would coincide with $FT \leftrightarrow PTE$ (within employment) flows. The second reason of failure to find full-time work would coincide with $U \leftrightarrow PTE$ flows (across employment/nonemployment states). The fluctuation in the within-employment flows and the lack of any fluctuation in the flows across PTE and U/N indicate that firm-level demand and the “slack work” reason are the primary source of cyclical variation in part-time employment for economic reasons.

To establish that within-employment flows fluctuate, examine the cyclical properties of flows between FT and PTE in Column 1 of Table 3. The flows of full-time employed to PTE are strongly countercyclical and 6.4 times as volatile as aggregate output. Meanwhile, flows from PTE to FT are strongly procyclical. In contrast with the cyclical movements in the $FT \leftrightarrow PTE$ flows, the across-employment/nonemployment flows of workers between PTE and U in Column 2 of Table 3 have very little correlation with aggregate output. The difference in the cyclical variation in the within-employment and across-employment/non-employment flows is obvious if we plot the probabilities of unemployed workers to both full-time and part-time for economic reasons. Figure 3 shows the variation in the $U \rightarrow FT$ flow (the full-time job finding probability) in comparison with the nearly constant $U \rightarrow PTE$ flow probability. If we suspected the popular narrative of the PTE worker as someone who failed to find a full-time job to be the source of the fluctuations in PTE , we would expect that in recessions, a higher fraction of the unemployed would move to PTE . While it is true that more workers in total arrive to PTE from U in recessions, this increase is entirely due to the size of the unemployment stock rather than an increase in the probability of a worker moving to PTE .⁷

To establish that the cyclical movements in the PTE rate are not just within-employment but specifically due to within-job hours changes, I use the dependent interview structure of

⁷As an additional check, I plot the shares of the stock of PTE workers by their status the previous month in Figure 14 in Appendix C. The majority of workers reporting to be in PTE in a given month come from voluntary employment (either full-time or part-time) or were in PTE the month before. The percentage of the PTE stock which comes from the inflow of nonemployed (both U and N) individuals is also constant over the business cycle.

Table 3: Cyclical Properties of flows between FT and PTE

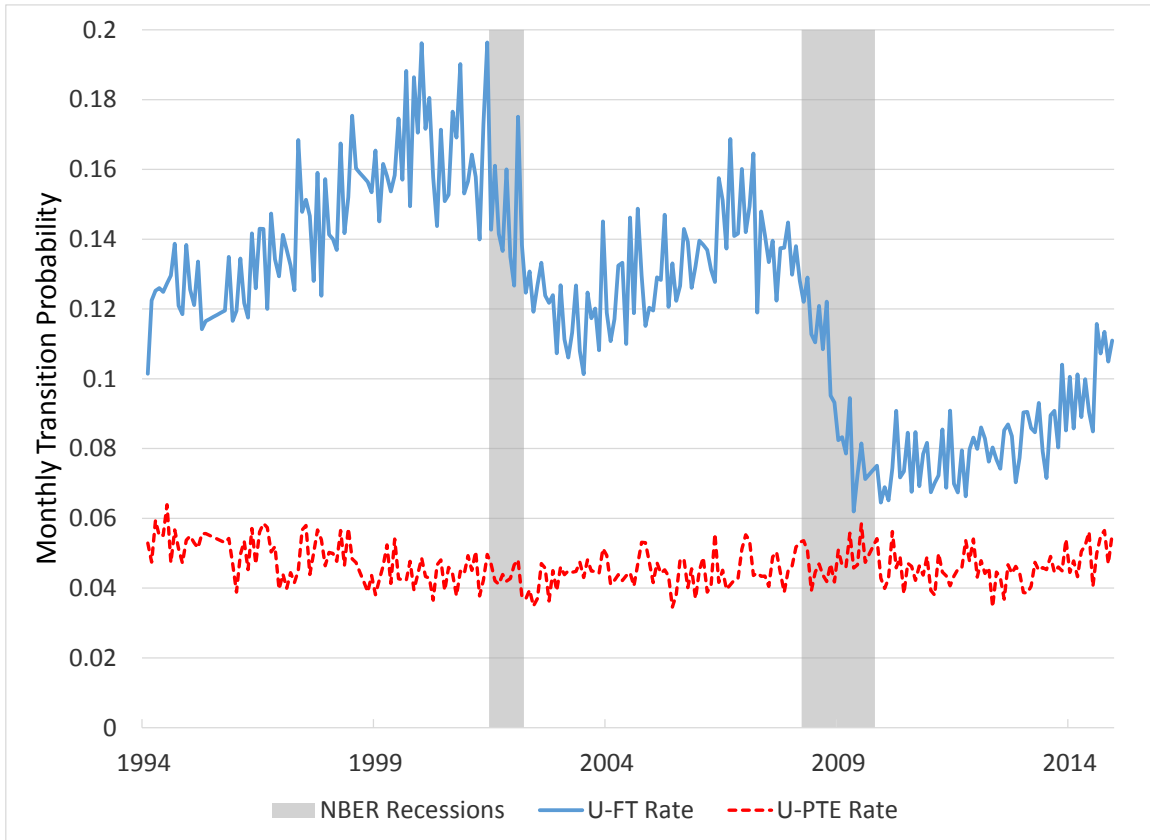
From	Column 1		Column 2	
	FT	PTE	PTE	U
To	PTE	FT	U	PTE
$std(x)$.150	.077	.092	.075
$corrcoef(Y, x)$	-.773	.855	-.255	-.245

Cyclical components of quarterly-averaged monthly series, logged and HP Filtered with $\lambda = 100,000$

the post-1994 CPS to classify which transitions in full-time and part-time status coincide with job changes. In months 2-4 and 6-8 of a respondent’s survey, the interviewer asks if the individual is still employed with the employer named in the previous month. As shown in Fallick and Fleischman (2004), this dependent interview makes it possible to track job-to-job transitions for workers who remain employed in consecutive months. I note which transitions between FT , PTN , and PTE correspond with a change in job. Especially with part-time and full-time status changes, one relevant concern is that the gain or loss of jobs for multiple job holders could be responsible for a substantial fraction of the $FT - PTE$ flow. Therefore, I include any change in the listed primary employer or any change in the number of jobs held for multiple job holders as my measure of a job transition. Figure 4 plots the separate transition probabilities of workers from FT to PTE while staying with the same employer for transitions to PTE while experiencing a job transition. Comparing the flow probabilities from FT to these states, it is clear that the majority of the $FT - PTE$ flow and its cyclical movements come from within-job changes in hours worked rather than from job-to-job transitions. Flow probabilities from PTE back to FT via the same job or a job transition (either a job-to-job transition or an increase to multiple jobs) are also similarly plotted in Figure 5.

It is also useful to observe the fraction of transitions which correspond with a job change. Looking at flows from FT to FT , PTN , and PTE in Figure 6, it is interesting to note that until the recovery from the Great Recession, the share of $FT - PTE$ transitions which come from a job change is declining. This decline is especially steep during the last recession.

Figure 3: Monthly Transition Probabilities out of Unemployment

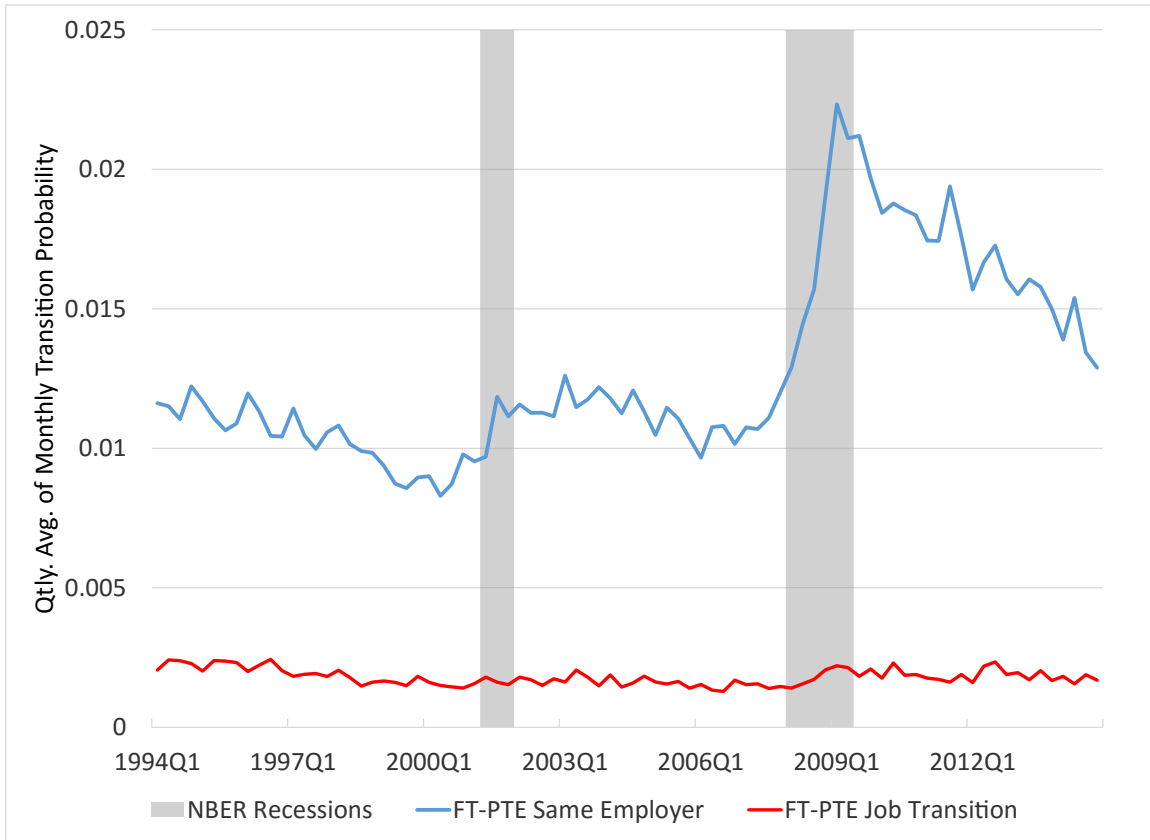


The solid blue line plots the fluctuating transition probability of a worker from unemployment to full-time work. The dashed red line plots the nearly constant probability of an unemployed worker moving to *PTE*.

Looking at Figure 7, the fraction of flows out of *PTE* that are due to job changes is also relatively small and declining. 80 – 90% of transitions from *PTE* back to full-time employment come from within-job. These figures also show that the majority of transitions into and out of *PTE* by individuals who remain in employment occur within-job. This fact is not specific to just the *FT* ↔ *PTE* flows. In fact, of all job transitions of employed workers into or out of *PTE* (excluding *PTE* – *PTE* flows with the same employer), 82% of all transitions occur within job.

One limitation of the CPS data is the inability to observe whether or not these movements of workers between employment states correspond with specific job transitions, especially for multiple job holders. While the CPS tracks the number of jobs for multiple job holders and job transitions are clearly identified for single job holders, The only job identity maintained

Figure 4: *FT – PTE* Transition Probability by Job Transition Status

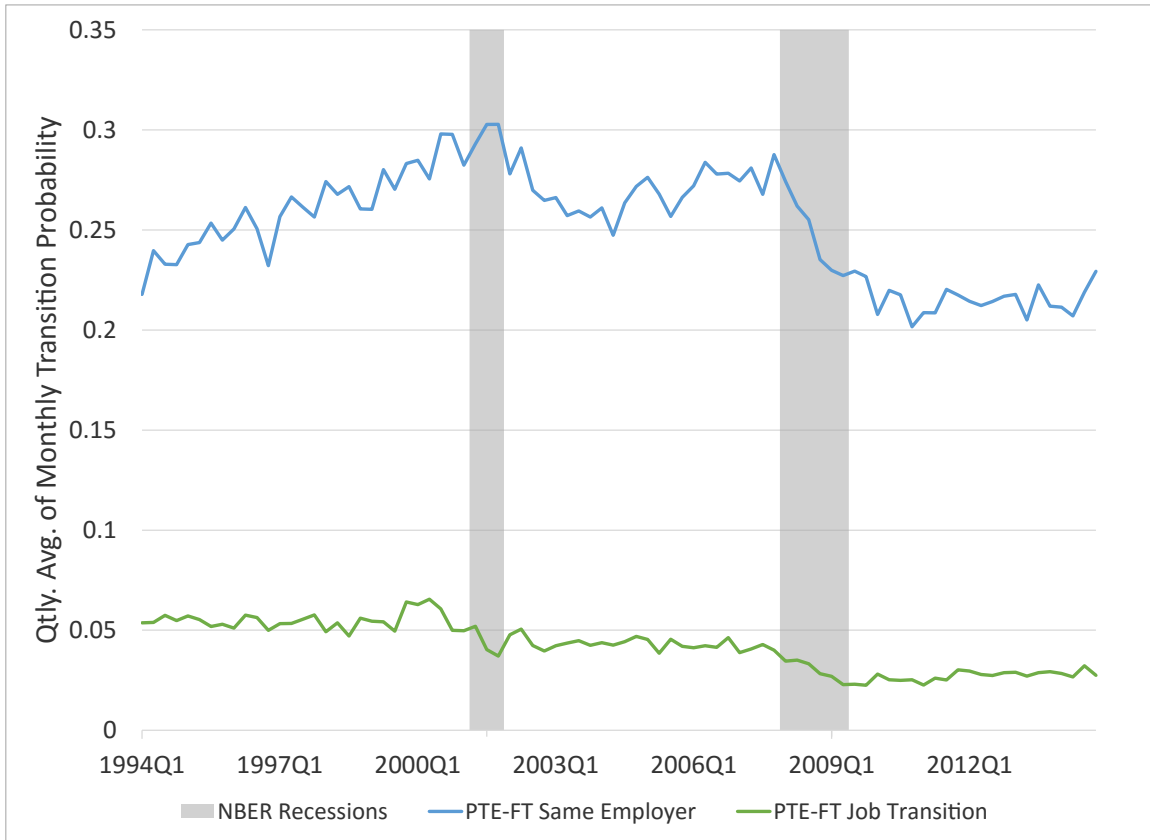


The blue line plots the quarterly averaged monthly probability of moving from full-time to part-time for economic reasons while remaining with the same employer. The red line plots the probability of the same transition while also experiencing a job transition.

is the primary job of the respondent. One cannot distinguish a job transition for a multiple-job holder who remains at their primary job if the number of jobs held remains constant in both months. For example, there is no way to tell if a multiple-job holder moved from *FT* to *PTE* because of a job-to-job transition in their secondary job. Fortunately, labor force participation data in the Survey of Income and Program Participants (SIPP) tracks all jobs held in each month, allowing for all job transitions to be tracked.⁸ I find that in the

⁸The SIPP follows about 60,000 individuals over a time-frame spanning 36-48 months, and provides monthly responses of workers' labor force participation, weekly hours worked, and reported reason for working part-time. There are several differences between the structure of the SIPP and CPS surveys which are noted in Appendix C.6. Although the exact equivalent to the definition of "At work part-time for economic reasons" cannot be reconstructed in the SIPP, I classify workers as full-time or part-time based on their self-response as part-time or full-time, and infer *PTE* workers from the reason they provide for being employed part-time. Due to the panel structure and the availability of job-specific data in the SIPP, I can observe whether a worker's change in status to or from *PTE* corresponds with a change in jobs, including the gain or loss of

Figure 5: *PTE* – *FT* Transition Probability by Job Transition Status

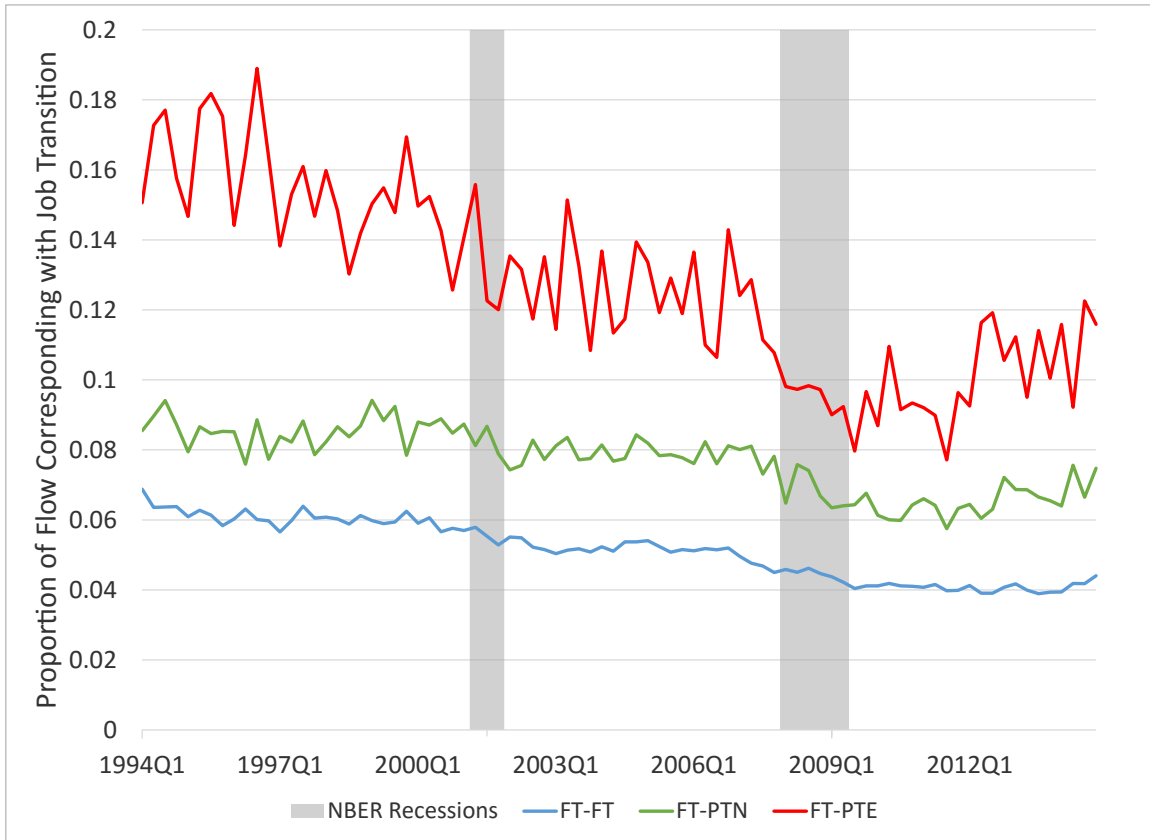


The blue line plots the quarterly averaged monthly probability of moving from part-time for economic reasons to full-time while remaining with the same employer. The green line plots the probability of the same transition while also experiencing a job transition.

1996 panel of the SIPP, The fraction of flows into and out of *PTE* which correspond to job changes are similar to those found in the CPS data. 75% of all flows into or out of *PTE* do not coincide with any job transition.

Before moving on to the model, it is important to establish that although the cutoff for defining part-time vs. full-time work is somewhat arbitrary at 35 hours per week, the actual hours worked by full-time and part-time employees reflect the notion that the part-time workweek is significantly shorter than full-time work. This is especially important to note for the case of part-time work for economic reasons, as it is possible that within-job movements between *FT* and *PTE* could be an artifact of firms reducing workers' hours only slightly one or multiple jobs.

Figure 6: *FT – PTE* Transition Probability by Job Transition Status



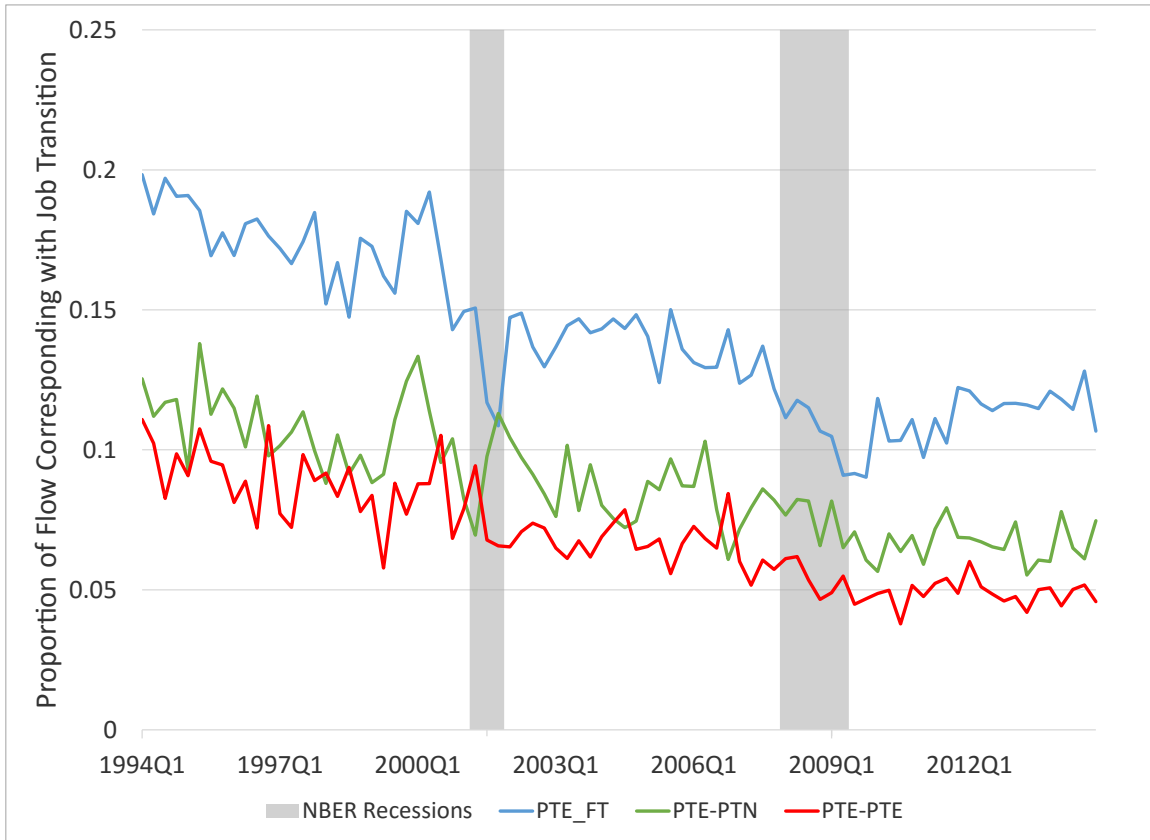
The quarterly averaged proportion of each transition probability out of *FT* which was accompanied with a job transition, defined as a change in the primary employer reported the previous month or a change in the number of jobs held if the respondent was a multiple job holder in either month.

below the 35 hour cutoff. This is not the case, however, as both part-time for economic reasons (23.11 hrs/wk) and noneconomic reasons (21.61 hrs/wk) work roughly half of full-time hours (44.55 hrs/wk).

3 A Model of Part-time Employment

To study the implications of firm-level changes in demand in the aggregate labor market over the business cycle, I build a model of part-time employment. I incorporate a part-time margin in a competitive search environment with large firms and costly recruitment. The model builds on the framework of Kaas and Kircher (2015), with the extension of the

Figure 7: *PTE* – *FT* Transition Probability by Job Transition Status



The quarterly averaged proportion of each transition probability out of *PTE* which was accompanied with a job transition, defined as a change in the primary employer reported the previous month or a change in the number of jobs held if the respondent was a multiple job holder in either month.

firm’s problem to account for a part-time employment decision. Firm size is determined by decreasing returns to scale production in labor, while firm growth is subject to search frictions and convex vacancy posting costs.⁹ Alternative frameworks incorporating firm size and search include the random search models of Elsby and Michaels (2013) and Moscarini and Postel-Vinay (2013) and the directed search model in Schaal (2012). These environments, however, all incorporate linear vacancy posting costs. The inclusion of convex recruitment costs produces rich firm dynamics over the life-cycle of the firm, rather than instantaneous firm growth as would occur under linear vacancy costs.

The model is in discrete time with a continuum of risk-neutral workers of mass one and

⁹Another way to determine firm size is through wage-posting and competition over workers as used in Moscarini and Postel-Vinay (2013).

an endogenous mass of firms. Workers and firms discount future payoffs at a rate $\beta < 1$. The timing of the model is as follows: first, aggregate productivity is revealed, new firms pay an entry cost, and idiosyncratic productivity for firms is revealed. Next, firms choose the fraction of their labor force to employ on a part-time basis, α . Firms then produce using their stock of employed labor, post contracts for new hires and vacancy postings, and choose the separation rate for workers. Firms exogenously exit at the end of recruitment with probability δ . Workers consume their wages, and unemployed or part-time employed workers consume leisure. Lastly, unemployed workers and vacancies are matched, and separations occur, changing the stocks of unemployment and employment for the next period.

3.1 Firms

Firms operate a decreasing returns to scale production technology. A firm's output in one period is $xzF(L)$ when utilizing a mass $L \geq 0$ of labor in production. F is twice differentiable, strictly increasing, strictly concave, and satisfies the Inada conditions. $x \in X$ is the firm's level of idiosyncratic productivity, and $z \in Z$ is aggregate productivity: X and Z are finite state spaces. Because workers are identical, and hours and bodies are assumed to be perfect substitutes, a firm produces the same output from utilizing one unit of labor in production, regardless of this unit coming from two part-time workers (working half-time) or one full-time worker. I assume that the firm only has the ability to employ any individual worker either at full-time or at half-time. The firm's part-time utilization is decided by choosing what fraction $\alpha \in [0, 1]$ of its labor force to employ at half-time (henceforth part-time), while the fraction $1 - \alpha$ of its labor force is employed full-time and supplies one unit of labor per worker.

Firms choose separation and the fraction of their labor force to employ part-time as a function of contingent histories of x and z . A firm of age j at time t experiencing history (x^j, z^t) is subject to an exogenous exit probability δ , and chooses a separation probability $s(x^j, z^t) \in [s_0, 1]$, and a fraction of its labor force to employ part-time, $\alpha(x^j, z^t) \in [0, 1]$. The

probabilities $\delta \geq 0$ and $s_0 \geq 0$ reflect the possibility of exogenous firm death and worker separation, respectively.

Firms choose history-contingent recruitment policies to hire workers. A firm with workforce L and productivity (x, z) that posts V vacancies pays recruitment costs $C(V, L)$. I adopt a constant-returns specification for the firm's vacancy cost function $C(V, L)$, as in Merz and Yashiv (2007):

$$C(V, L) = \frac{c}{1 + \gamma} \left(\frac{V}{L} \right)^\gamma V.$$

With this specification, the average cost per vacancy for a firm depends on the firm's vacancy rate (V/L) . The flow cost per vacancy is equal for all firms recruiting the same percentage of their current labor force.

3.2 Matching and Contracts

The labor market features competitive search, under which firms compete for workers by posting long-term contracts. Unemployed workers direct their search to those postings that yield the highest expected utility, taking the fact that better contracts will have a lower probability of matching into account. With a standard matching function, at each contract type there is a corresponding queue length of λ unemployed per vacancy, yielding a matching probability for the firm's vacancy of m . If a firm is to fill a vacancy with probability m , it must offer a contract that attracts $\lambda(m)$ workers. With the usual assumptions on the matching function, the function $\lambda(m)$ is the inverse of the matching function, and satisfies $\lambda(0) = 0$, $\lambda'(0) \geq 1$, and $\lambda'(1) = \infty$. The worker's probability of matching is thus $m/\lambda(m)$.

Firms offer contracts to each cohort of workers recruited in a given period. These contracts specify a sequence of policies which apply to each worker in this cohort for every contingent history so long as the match exists. If the state of a contract offered by an age j firm in a particular history is defined as $\tau = \{j, (x^j, z^t)\}$, then a contract is denoted as:

$$\mathcal{C}(\tau) = (w_f(\tau), w_p(\tau), \alpha(\tau), \phi(\tau))$$

The retention probability $\phi(\tau)$ is the firm's exit probability δ multiplied by the probability of match separation $s(\tau)$, so that the probability of separation for a worker is $1 - \phi(\tau)$. $\alpha(\tau)$ specifies the probability of a worker being placed on part-time work. The wage is a function of full-time or part-time employment: $w_f(\tau)$ or $w_p(\tau)$, respectively.¹⁰ Due to risk-neutrality, workers simply value the expected present value of being employed in a contract, $W(\mathcal{C}(\tau))$.¹¹

3.3 The Worker's Problem

A worker can be employed full-time, contributing one unit of time to labor, or part-time at 1/2 unit of labor. If unemployed, the worker receives utility $b \geq 0$, representing leisure or home production. Similarly, a part-time employed worker receives utility ℓb , where $\ell \in (0, 1)$ is the fraction of leisure or home production gained in part-time. The parameter ℓ reflects the fact that part-time work frees up only a fraction of time for leisure, as well as the possibility that some portion of b could come from transfers or unemployment insurance benefits that would not be available to a part-time worker.

Let $U(z^t)$ be the utility value of an unemployed worker in history z^t , and $W(\mathcal{C}(\tau))$ be the present expected value of employment in contract $\mathcal{C}(\tau)$ to a worker before production occurs. Because utility is linear, the worker treats the probability of part-time work as a lottery. The employed worker's value function satisfies the recursive equation:

$$\begin{aligned}
 W(\mathcal{C}(\tau)) = & [w_f(\tau)(1 - \alpha(\tau)) + w_p(\tau)\alpha(\tau)] + \alpha(\tau)(\ell b) \\
 & + \beta \{ (1 - \phi(\tau))\mathbb{E}_{z'}U(z^{t'}) + \phi(\tau)\mathbb{E}_{\tau'}[W(\mathcal{C}(\tau'))] \}. \tag{1}
 \end{aligned}$$

An unemployed worker's search problem involves maximizing the expected utility gain of employment, taking the matching probability and value of each contract into consideration.

¹⁰In this case, wage is total wage per period rather than wage per unit time worked.

¹¹Note that these contracts are general, and may be binding for both workers and firms, in that separation and part-time probabilities specified in the contract may bind either party. The assumption of commitment on either side may be relaxed in some cases. In general, however, the timing of payoffs to the worker depends on the ability to commit to certain actions.

Potential contracts are observed, and parameterized by the tuple $(m, \mathcal{C}(\tau))$. Knowing that a contract yields a probability of matching for the worker of $m/\lambda(m)$, the worker's search value is:

$$\rho(z^t) = \max_{(m, \mathcal{C}(\tau))} \frac{m}{\lambda(m)} (1 - \delta) \beta \mathbb{E}_{\tau'} [W(\mathcal{C}(\tau')) - U(z^{t'})], \quad (2)$$

which reflects the expected probability of matching with a firm offering contract $\mathcal{C}(\tau)$ multiplied by the expected gain in the worker's value function derived from being employed with that contract during the next period. The Bellman equation for the unemployed worker then satisfies

$$U(z^t) = b + \rho(z^t) + \beta \mathbb{E}_{z^{t'}} U(z^{t'}). \quad (3)$$

The unemployed worker receives constant flow utility b from leisure or unemployment benefits, and the option value of searching, $\rho(z^t)$. Since workers can direct their search to different contracts, their flow value from unemployment must be equal in any market that attracts workers. This implies that $\rho(z^t)$ is common to workers in any submarket; hence, $\rho(z^t)$ determines the contract value the firm must post to fill a vacancy with a positive probability m .

3.4 The Firm's Problem at Age 1

The firm's problem involves the recruitment of new workers through the posting of contracts for new vacancies, and the commitment to past contracts offered to its current labor force. The firm of age j in history (x^j, z^t) takes as given its current stock of workers hired up to the current period, $(L_i)_{i=0}^{j-1}$, and the contracts signed by previous labor cohorts, $(\mathcal{C}_i)_{i=0}^{j-1}$. The firm's separation rate and retention probability must satisfy $s(\tau) = (1 - \phi(\tau))/(1 - \delta)$ in order to avoid violating its prior commitments to past labor cohorts, and to be consistent with exogenous probabilities of separation and firm death. The firm then decides to post a contract \mathcal{C}_j in V vacancies. The contract \mathcal{C}_j achieves the firm's desired matching probability

m , taking the worker's value of search $\rho(z^t)$ as given. To simplify, I describe the problem of a firm of age 1 with a single cohort of existing workers. The complete firm's problem and definition of competitive equilibrium appears in Appendix A.

Consider a firm of age 1 in history (x^1, z^t) . Letting $\tau = \{1, (x^1, z^t)\}$, the problem of the age 1 firm is:

$$J[(\mathcal{C}_0), (L_0), \tau] = \max_{m, V, \mathcal{C}_1} x_1 z_t F[\tilde{L}(\tau)] - \mathcal{W}(\tau) - C(V, L_0) + \beta(1 - \delta)\mathbb{E}_{\tau'} \{J[(\mathcal{C}_{0,1}), (L_{0',1}), \tau']\} \quad (4)$$

s.t.

$$\tilde{L} = L_0(1 - \alpha_0(\tau)/2), \quad L'_1 = mV, \quad L'_0 = L_0 \frac{\phi_0}{(1 - \delta)} \quad (5)$$

$$\mathcal{W}(\tau) = L_0[(1 - \alpha_0(\tau))w_{f0}(\tau) + \alpha_0(\tau)w_{p0}(\tau)] \quad (6)$$

$$\rho(z^t) = \frac{m}{\lambda(m)}(1 - \delta)\beta\mathbb{E}_{\tau'}[W(\mathcal{C}(\tau')) - U(z^t)] \text{ if } m > 0. \quad (7)$$

The firm enters the period with an existing labor force L_0 hired under contracts \mathcal{C}_0 . The firm produces using \tilde{L} in (5) and pays a wage bill \mathcal{W} in (6), with α and $\{w_f, w_p\}$ being consistent with contract \mathcal{C}_0 . The firm chooses contracts \mathcal{C}_1 to post in V vacancies at a cost $C(V, L)$. It chooses a matching probability m through the value of the contracts posted, knowing that achieving m requires a queue length of $\lambda(m)$ per vacancy. The minimum utility a contract \mathcal{C} must promise to attract a queue length of $\lambda(m)$ per vacancy comes from the worker's participation constraint, (7). At the end of the period, the firm's current cohort separates at rate $s = \frac{\phi}{1 - \delta}$ leaving $L'_0 = L_0(1 - s)$ remaining in that cohort. The firm then gains a new cohort $L'_1 = mV$ from recruitment (5). This problem generalizes to the firm of age j , where each cohort and contract is indexed by the age of the firm at recruitment. The total labor force and wage bill are similar, except that they consist of the sum of all cohorts and their respective contracts. Lastly, the firm's policies must be consistent with its commitment to past cohorts.¹²

¹²Section A provides the general problem of the firm of age j , and the description of the free entry condition

Free entry of firms implies that the expected value of an entrant firm (before realizing x and with labor force $L = 0$) is less than or equal to the entry cost $K(z_t)$. Letting $\tau_0 = \{0, x_0, z_t\}$:

$$\sum_{x_0 \in X} \pi(x_0) J[0, 0, \tau_0] \leq K(z_t). \quad (8)$$

If entry is positive, then this condition holds with equality.

3.5 Planner's Problem

The competitive equilibrium of the decentralized problem involves history-contingent and cohort-dependent policy functions for firms. Solving for the decentralized competitive equilibrium requires the tracking of the composition of worker cohorts within firms of different sizes and age-groups in all potential aggregate and idiosyncratic histories. The welfare theorems apply in this framework, so it is possible to solve an equivalent planner's problem. I show that the planner's problem simplifies to solving the maximization problem of the value of an individual firm given its current state, independent of cohorts and histories.

The sequential planner's problem is to maximize expected discounted output net of firm entry costs, opportunity costs of work, and recruitment and operating costs of the firm, subject to the constraint of having a unit mass of workers. Let $\psi(z^t)$ be the probability of aggregate history z^t given the transition matrix for aggregate states ψ , and $N(x^j, z^t)$ denote the measure of firms of a particular history. The planner's problem is:

$$\begin{aligned} & \max_{s, \alpha, V, M, N_0} \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) N_0(z^t) \right. \\ & + \sum_{j \geq 0, x^j} N(x^j, z^t) \left[x_j z_t F(L(x^j, z^t)(1 - \alpha_\tau(x^j, z^t)/2)) \right. \\ & \left. \left. - b(1 - \ell\alpha(x^j, z^t)) L(x^j, z^t) - C(V(x^j, z^t), L(x^j, z^t)) \right] \right\} \quad (9) \end{aligned}$$

and definition of a competitive equilibrium.

subject to

$$N(x^{j+1}, z^{t+1}) = (1 - \delta)\pi(x_{j+1}|x_j)\psi(z_t + 1|z_t)N(x^j, z^t) \quad \forall(x^j, z^t) \quad (10)$$

$$L(x^{j+1}, z^{t+1}) = [1 - s(x^j, z^t)]L(x^j, z^t) + m(x^j, z^t)V(x^j, z^t) \quad \forall(x^j, z^t) \quad (11)$$

$$N(x_0, z^t) = \pi_0(x_0)N_0(z^t) \geq 0 \text{ and } L(x_0, z^t) = 0 \quad \forall z^t \quad (12)$$

$$\sum_{j \geq 0, x^j} N(x^j, z^t)[L(x^j, z^t) + \lambda(m(x^j, z^t))V(x^j, z^t)] \leq 1 \quad \forall z^t. \quad (13)$$

Proposition 1 *A Competitive search equilibrium is socially optimal.*

Proof See Appendix B 3

The planner's problem specifies history-contingent, cohort-contingent policies as in the competitive equilibrium. There exists, however, a solution to the planner's problem where policies are not only cohort-independent, but also independent of idiosyncratic and aggregate histories. Further, the planner's problem can be rewritten as the sum of recursive problems maximizing the social surplus of each firm. The value of a worker to the planner is given by the Lagrange multiplier on the aggregate feasibility constraint (13). Given a vector of multipliers for the aggregate resource constraint (13), $M = (\mu_1, \dots, \mu_n)$ for each $z \in (z_1, \dots, z_n)$, the social value of a firm with productivity x and z and labor stock L satisfies the following Bellman equation:

$$\begin{aligned} G(L, x, z; M) = & \max_{s, \alpha, V, m} xzF(L(1 - \alpha(x, z)/2)) - bL(1 - \ell\alpha(x, z)) \\ & - \mu(z)[L + \lambda(m(x, z))V(x, z)] - C(V(x, z), L) \\ & + \beta(1 - \delta(x, z))\mathbb{E}_{x', z'} G(L', x', z'; M) \end{aligned} \quad (14)$$

subject to

$$L' = (1 - s)L + mV, \alpha \in [0, 1], s \in [s_0, 1], m \in [0, 1] \text{ and } V \geq 0.$$

The result that the planner's problem is cohort-independent follows from the fact that workers are identical and have linear utility, so they are indifferent about the time-path of payment. Thus, the planner gains nothing by specifying cohort-specific policies. Further, any sequence of history-contingent policies in a decentralized equilibrium can be reproduced by the planner using policies that only depend on current states of idiosyncratic and aggregate productivity. This is due to the fact that the solution to the planner's maximization problem for the social surplus of a firm dictates the planner's optimal policy for the firm as a function of firm size, current productivity, and the social value of a worker, $\mu(z)$. If entry is positive, then $\mu(z)$ is pinned down by the entrant firm value that satisfies the free-entry condition (which is only a function of the aggregate state z_t) with equality. Since the social value of a worker is identical across all firms, and the social value of any firm can be solved for any size L and productivity realization x, z given $\mu(z)$, the planner's problem for each firm can be solved given $\mu(z)$. Thus, entry and firm-level policies and value-functions can be solved where the only aggregate state variable for the planner is the current realization of z_t .

Efficient entry of firms requires that the entry condition is satisfied with equality:

$$\sum_{x \in X} \pi_0(x) G(0, x, z; M) = K(z). \quad (15)$$

Proposition 2 (a.) *Suppose that a solution of (14) and (15) exists with associated allocation $\mathbf{A} = (\mathbf{N}, \mathbf{L}, \mathbf{V}, \mathbf{m}, \mathbf{s}, \alpha)$ satisfying $N(z^t) > 0$ for all z^t . Then \mathbf{A} is a solution to the sequential planning problem (9).*

(b.) *If $K(z)$ and b are sufficiently small and if $z_1 = \dots = z_n = \bar{z}$, equations (14) and (15) have a unique solution (G, M) . If the transition matrix $\psi(z_j|z_i)$ is strictly diagonally dominant and if $|z_i - \bar{z}|$ is sufficiently small for all i , equations (14) and (15) have a unique solution.*

Proof See Appendix B 4

Conversely, the solution of the planner’s problem with history-independent policies coincides with a solution of the competitive equilibrium. There are many decentralizations that yield a payoff-equivalent allocation to the planner’s problem, though not necessarily the same allocation as the planner. This is due to the linearity of workers’ utility so that the time-path of payments doesn’t matter to the worker. Also important is the redundancy of cohort-specific policies for the planner. One simple example of a decentralization of the planner’s allocation is if firms offer contracts which provide a “constant utility” contract by offering a constant full-time wage and a constant part-time wage. The firm provides the same utility for part-time workers as it does for full-time work after accounting for the utility gained in leisure during part-time. Workers commit to employment until the firm destroys the match, and they are indifferent between full-time or part-time employment. In general, the nature of the contract and the ability for either workers or firms to commit to specific actions will affect the time-path of payments.

4 Dynamics of the Firm

The model’s outcomes for firm-level labor demand through hiring, firing, and layoff can be illustrated through the optimal decisions of the planner’s problem for a firm. These conditions highlight that firm expansion occurs gradually over time through variations in the number of vacancies posted by the firm and the matching probability of each vacancy.

The planner’s problem can be solved given a vector $M = (\mu_1, \dots, \mu_n)$ of multipliers on the resource constraint (13), which correspond to the planner’s value of a worker in n aggregate states. The first-order conditions determine optimal policies for $s, \alpha, m,$ and V for a given level of productivity and firm size L . The first order conditions for m and V relate the inter-temporal and intra-temporal tradeoffs of recruitment intensity for firms. The optimal recruitment policy in a stationary equilibrium features a declining match probability as L increases to the firm’s optimal size.

Suppressing the state (x, z) for policy functions, the first-order condition with respect to m gives:

$$\beta(1 - \delta)\mathbb{E}_{x', z'} \frac{\partial G(L', x', z')}{\partial L'} = \mu(z)m\lambda_m(m). \quad (16)$$

where $L' = L(1 - s_0) + mV$ if m is positive. Similarly, the first-order condition with respect to V gives:

$$\beta(1 - \delta)\mathbb{E}_{x', z'} \frac{\partial G(L', x', z')}{\partial L'} = \mu(z)\lambda(m) - C_V(V, L) \quad (17)$$

Combining (16) and (17) gives the intra-temporal optimality condition for hiring, describing the trade-off to the planner of increasing vacancies and increasing the matching probability for the firm. Intuitively, the planner equates the marginal cost to the firm of hiring another worker by opening more vacancies with the marginal cost of hiring an additional worker by increasing the matching probability of vacancies through more valuable contracts:

$$\mu(z)[m\lambda_m(m) - \lambda(m)] = C_V(V, L) \quad (18)$$

Using $\frac{\partial G(L', x', z')}{\partial L'}$ via the envelope theorem and (16), we can get the inter-temporal optimality condition, which governs hiring intensity of the firm over time.

$$\begin{aligned} \frac{\partial G}{\partial L'} &= x'z'F_{L'}(L'(1 - \alpha'/2)) - b(1 - \ell\alpha') - \mu(z') \\ &\quad - C_L(V', L') + (1 - s')\mu(z')\lambda_m(m') \end{aligned} \quad (19)$$

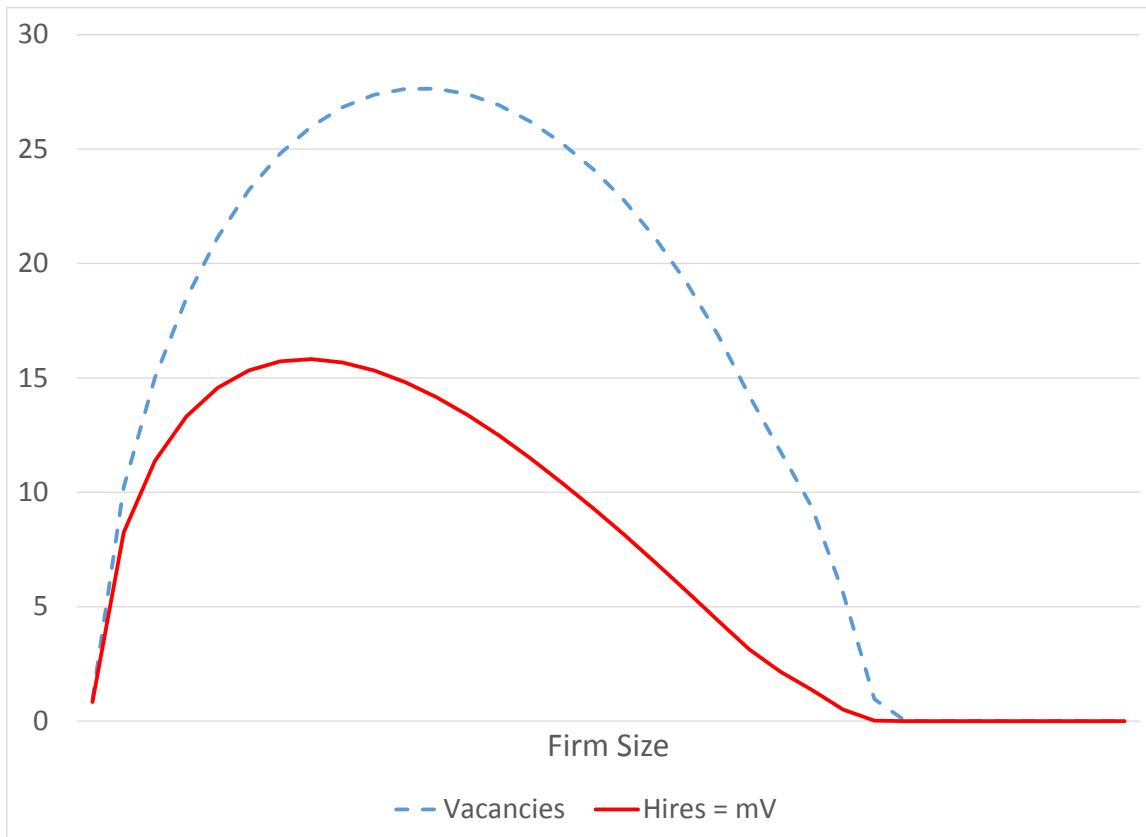
The planner hires if:

$$\beta(1 - \delta)\mathbb{E}_{x', z'} \frac{\partial G(L(1 - s_0), x', z')}{\partial L} > \mu_i\lambda'(\underline{m}),$$

where \underline{m} is the value of m for which $\lambda(m) = 0$. If this inequality is satisfied, (18) characterizes the optimal vacancy postings $V^*(m(L, x, z))$ for a given matching probability m which satisfies (16) with (19) substituted inside the expectation term.

Figure 8 shows the vacancies posted and number of hires (mV) for a firm of a given productivity level as a function of its current workforce. The increasing gap between the number of vacancies posted and the number of hires with labor L reflects the declining matching probability used by the firm as it reaches maturity. Since vacancy posting costs are convex, the firm spaces hiring over time.

Figure 8: Vacancies and Hiring Policies over the Life of a Firm.



Vacancy posting and hiring policies of a firm as a function of current firm size. Vacancies are plotted with the blue dashed line. Total hires per period (in red) is the product of vacancies and the matching probability m per vacancy, which is declining in size.

The optimal choice of α and s can also be solved from the first-order conditions of the firm's social surplus problem. The first-order condition with respect to α is:

$$xzF_L(L(1 - \alpha/2)) - 2\ell b = 0. \quad (20)$$

An alternative way to write this condition is to consider the planner's (firm's) ideal labor

utilization today, \hat{L} . The optimal labor utilization solves:

$$\hat{L} = F_L^{-1}\left(\frac{2\ell b}{xz}\right)$$

. Since F_L is positive and decreasing, its inverse is also decreasing. Thus, labor utilization \hat{L} is increasing in productivity xz and decreasing in the share of leisure gained by moving from full-time to part-time work, ℓb . Intuitively, if the marginal product of labor in production today is high, the planner wants to allocate worker time to production and away from leisure. When the current marginal product of labor is low enough, the planner would like to move workers to part-time as the gains from added leisure are higher than the gains in output for the marginal worker. Writing \hat{L} in terms of L and α and assuming an interior solution, the optimal part-time usage α^* is characterized by:

$$\alpha^* = 2 \left(1 - \frac{F_L^{-1}\left(\frac{2\ell b}{xz}\right)}{L} \right)$$

.
 If the optimal labor utilization \hat{L} is outside of the interval $(L/2, L)$, then α^* is a corner solution. When interior, part-time use is increasing in the leisure gained from part-time work, ℓb . Part-time use is decreasing in productivity, xz . We can also see that part-time usage is increasing in L , all else equal. It should be noted that the part-time decision is independent of future values of the firm, but the current size L for the firm is a function of past size and productivity as well as the future value of these workers to the firm. Thus, actual part-time utilization by firms depends on the difference between their actual size and their ideal current size, and it is not clear that part-time usage will be increasing or decreasing in firm size categories in the stationary distribution.

The first-order condition with respect to s is:

$$\beta(1 - \delta)\mathbb{E}_{x,z} \frac{\partial G(L', x', z')}{\partial L'} = 0. \quad (21)$$

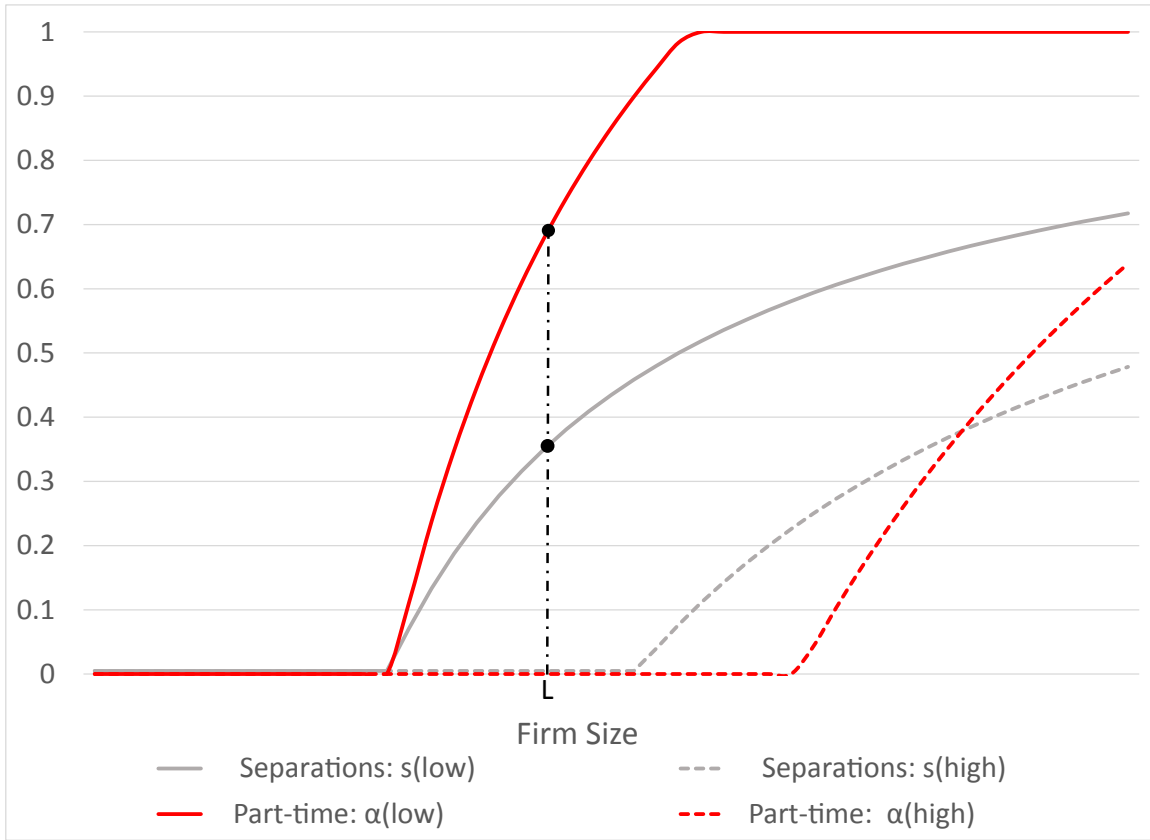
Again, if the solution is interior then (21) holds with equality. If the expected value of labor tomorrow is high, then the planner will not separate with workers.

From the first order conditions for α and s , the planner will utilize part-time labor when the value of current productivity is low. If the marginal value of its future labor force is also low, the planner will separate with workers at the end of the period.

Figure 9 shows the part-time probability and separation rate for a firm at two different productivity levels (high and low). The part-time employment rate (red line) and separation probability (light gray line) of the firm are the two upward-sloping lines at each productivity level. The slope of the part-time percentage is steeper than the separation rate. For a given size, the part-time utilization rate is increasing faster than the separation rate as the size of the shock increases. In general, the part-time rate can be positive even if the firm is not separating (it may even be expanding), depending on parameters. A high productivity firm initially at size L which realizes a low productivity shock would have policy functions of the solid lines in Figure 9. The black points on the part-time and layoff lines indicate the firm's choice of part-time usage that period, as well as the fraction of its workforce it will layoff at the end of the period. Upon realization of the shock from high to low productivity, the firm would use part-time employment to decrease its labor utilization in the current period. After production, the firm would separate with 35% of its workers, starting the next period with 65% of its original labor force.

While the policy functions and first-order conditions give intuition on the factors that affect firm-level decisions for employment and part-time utilization, the actual dynamics of the firm requires tracking of a firm's state variable over time. The firm's state variables (L, x, z) move according to the transition processes for productivity and the law of motion

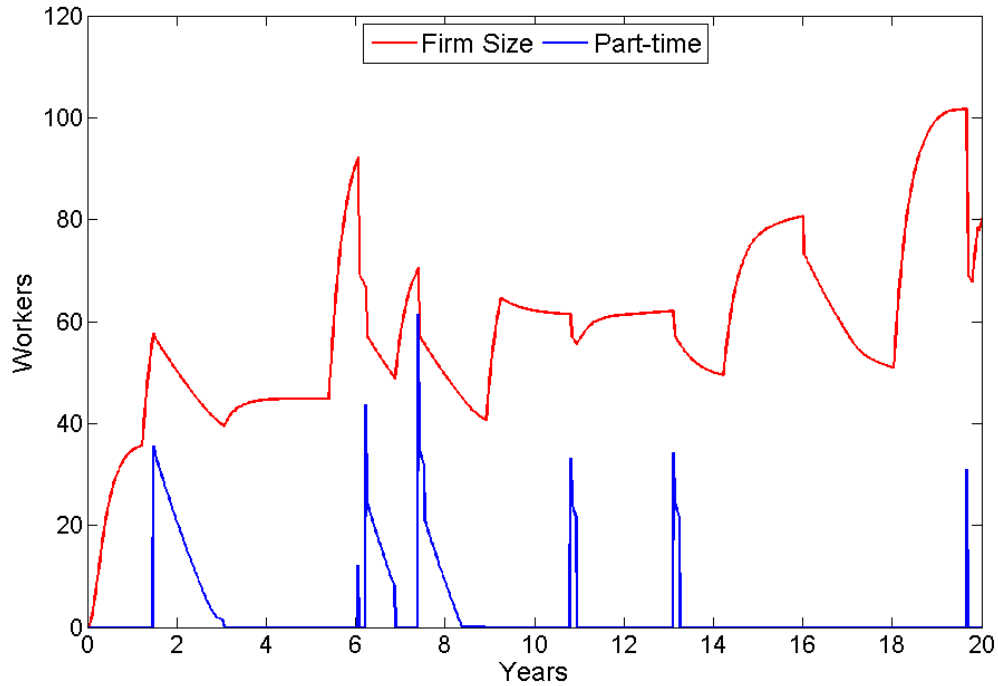
Figure 9: Part-time and Separation Rates in Response to a Productivity Shock



Dashed lines are the policies of a high productivity firm. Solid lines are the firm's policies after a low productivity shock. The black vertical line and dots indicate the choice of layoffs and part-time usage of a high productivity firm of size L after it receives a low productivity shock.

for firm size ($L' = L(1 - s) + mV$) dictated by the policy functions. In Figure 10, I plot the simulation of an individual firm subject to idiosyncratic and aggregate shocks over a 20 year period using parameters from the calibrated model. The firm grows upon entry and fluctuates with realized productivity shocks. Steep declines in the firm's size are the result of layoffs, while steady declines result from inaction in hiring or firing by the firm due to the exogenous separation of workers from the firm (at rate s_0). It is clear from the figure that part-time employment is volatile at the firm level and is used in conjunction with layoffs.

Figure 10: Dynamics of a Simulated Firm



The red line plots the firm's size L . The blue line plots the number of workers employed part-time, αL . Increases in firm size result from hiring, while sharp declines in size indicate layoffs. Smooth declines in firm size result from firm inaction and exogenous separations.

5 Quantitative Exercises

5.1 Calibration

To calibrate the model, I first match the model to moments of the aggregate labor market with idiosyncratic productivity shocks, holding the realizations of aggregate shocks at their average value. Next, I set productivity, entry, and exit parameters to match the firm size and age distribution. The idiosyncratic shock process matches the average separation rate and the share of employment at firms with little to no employment growth. The aggregate shock process matches the quarterly autocorrelation and standard deviation of average labor productivity. Table 5 displays the values of parameters and the targets they match.

Production I use a decreasing returns to scale function in labor:

$$xzF(L) = x_0x_1zL^\eta$$

where $\eta = 0.7$ is set to target a labor share of income of 0.66. Productivity of a firm is composed of an aggregate component z and idiosyncratic productivity $x = x_0x_1$. Idiosyncratic productivity is a product of a fixed component x_0 which is drawn upon firm entry and a transitory component x_1 .

Matching Function and Vacancies The matching function is of the form

$$m(\lambda) = (1 + k\lambda^{-r})^{(-1/r)},$$

where λ is the ratio of unemployed to vacancies and $m(\lambda)$ is the firm's probability of filling a vacancy. The cost function for vacancy postings is increasing in the vacancy rate of a firm, $\frac{V}{L}$:

$$C(V, L) = \frac{c}{1 + \gamma} \left(\frac{V}{L} \right)^{(1+\gamma)} L$$

where $\gamma = 2$ as in Kaas and Kircher (2015), so that the recruitment cost is cubic.

The model period is set to weekly, so the matching function parameter k is set to target a weekly job-finding rate of 0.129 (corresponding to a monthly rate of 0.45 as in Shimer (2005)) at the steady-state queue length. The parameter r is set to match an elasticity of the job-finding rate with respect to the tightness ratio ($\frac{V}{U}$) of 0.72. With multiple vacancies per firm and nonlinear posting costs, the rate of vacancy posting must be determined. The job-filling rate for vacancies is set to 0.3 to match the observed monthly vacancy yield of 1.3 as reported in Davis et al. (2013).¹³ The parameter c in the vacancy cost function is set to

¹³The vacancy yield is the ratio of jobs created in a month to the observed stock of vacancies observed at a point in that month. Thus, a vacancy yield of 1.3 reflects that there are more hires in a month than the observed instantaneous stock of vacancies. This reflects time-aggregation as additional vacancies are posted as jobs get filled, and the vacancy yield is determined by the rate of vacancy postings and the job-filling rate.

match this weekly job-filling rate of 0.3. In steady state, the queue length is the ratio of the job-filling and job-finding rate, yielding a steady-state queue length of $\lambda = 2.326$.

Home Production and Part-time The benefit from home production b is set to match 70% of the average wage, corresponding with the calibrated value of non-market work in Hall and Milgrom (2008). To calculate the replacement ratio, I consider the special case of a constant wage per period for full-time and part-time workers. That is, the wage contract is a pair of constants $\{\bar{w}_f, \bar{w}_p\}$ for the duration of the match. The share of leisure gained by part-time workers, ℓ , is used to target a steady-state fraction of employment working part-time of 5%.

Permanent Firm Types and the Size/Age Distribution of Firms What remains to be determined is the set of productivity parameters xz and their shock process. To match the broad size distribution of firms and the fact that a small fraction of very large firms retains a large share of total employment, the permanent component of idiosyncratic productivity x_0 is set to match the size distribution of firms. To match the firm share and employment distribution for 5 classes, x_0 takes on 5 values at entry, with entry share σ . The exit probability of firms, δ , is exogenous and also dependent on size classification. The vector δ is chosen by matching the observed job destruction rate for closing firms in each size class from the Bureau of Labor Statistic's Business Employment Dynamics data for 1992-2011. Using permanent firm types characterized by permanent productivity level x_0 , entry share σ , and exit probability δ given in Table 4, the model can closely match the firm size and age distribution in the data, as seen in Figure 11.¹⁴ The vector x_0 matches the employment shares of firms in each size category, while σ and δ determine the share of firms in each size category.

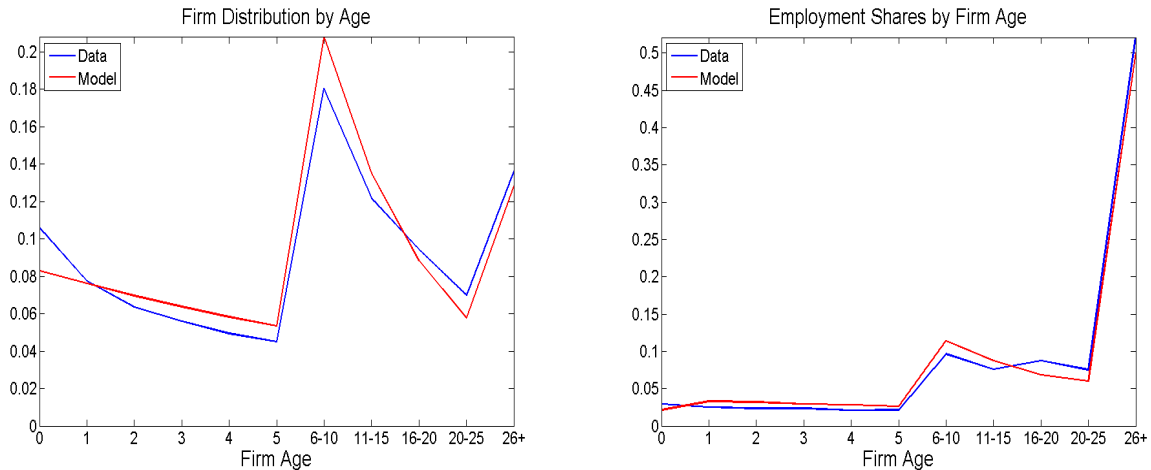
¹⁴Table A.3 in Appendix A.3 gives the share of firms and employment across firm size categories in the data and the model.

Table 4: Parameters for Matching the Size and Age Distribution of Firms

Size class	Permanent Productivity Types					Target/Source
	1-49	50-249	250-999	1k-9,999	10k+	
(x_{0i})	0.363	0.736	1.168	2.03	4.138	Emp. Shares of firm size categories
(σ_i) , %	98.82	1.00	0.153	0.025	0.002	Firm Shares in size categories
(δ_i) , %	1.71	0.27	0.16	0.088	0.016	BED Data on job destruction by firm size

Parameter values for permanent firm types used for matching the size and age distribution of firms.

Figure 11: Distribution of Firms by Age (in years): Firm Shares and Employment Shares



Cross-sectional relationship between firm age (in years) and firm shares/employment shares. The blue lines are from Business Dynamics Statistics data by the Census Bureau for 2005. Model statistics are in red

Idiosyncratic Shocks The shock process x_1 is evenly spaced between $[1 - \bar{x}, 1 + \bar{x}]$ and is redrawn with probability π each period. This shock process matches the fact that many firms in the data experience little to no net job growth in a given quarter. The idiosyncratic shock parameters match the average monthly separation rate and the share of employment at firms with monthly growth rates between -2% and 2% . The exogenous separation rate s_0 is set to match a monthly quit rate of 2% per month from the Job Openings and Labor Turnover Survey (JOLTS) data.¹⁵

¹⁵The calibration strategy is taken from Kaas and Kircher (2015) to facilitate comparison with their model when the part-time decision of firms is absent.

Aggregate Shocks The aggregate shock process affecting z is a mean-reverting Markov process as in Appendix C of Shimer (2005). The aggregate shock process is parameterized by a persistence parameter ψ and range $[\underline{z}, 2 - \underline{z}]$. The parameters $(\psi, \underline{z}) = 0.015, 0.95$ are chosen to produce a quarterly standard deviation and autocorrelation of productivity shocks of $\sigma_z = 0.015$ and $\rho_z = .76$. The remaining parameter to be set is the entry cost $K(z)$. The entrant firm’s value function is homogeneous in the vector $\{x_0, b, c, \mu, K\}$ in steady-state, so the stationary value of parameter $K(z)$ can be normalized arbitrarily. Tractability requires that there is positive entry of firms in every state, so that the value of μ for all workers is determined by the value at entrant firms. I allow $K(z)$ to vary with the aggregate state so that job creation at entrant firms is stable over the business cycle to ensure positive entry in all aggregate states.

I numerically solve for the values of $\mu, x_0, \ell, c, \bar{x}$, and π that minimize the target moments in steady-state (holding z at its average level). These moments are the job-finding rate, share of firms in each size category and share of employment by size categories, separation rate, ratio of b to average wages, and the employment share at firms with growth rate $< \pm 2\%$.

Table 5: Parameters in Benchmark Calibration

Parameter	Value	Target/Source
β	0.999	Annual interest rate 5%
b	0.1	70% of average wages
ℓ	0.63	Average PTE share of employment = 5%
η	0.7	Labor share of income = 0.66
k	6.276	job-finding rate (JFR) = 0.129
r	1.057	Elasticity of JFR wrt tightness ratio = 0.72
c	11.44	Vacancy-filling rate = 0.3/Vacancy yield = 1.3 (Davis et al., 2013)
γ	2.0	Cubic vacancy cost function (Kaas and Kircher, 2015)
s_0	0.0048	Monthly quit rate = 0.02
π_x	0.933	Share of employment in low-growth firms = 66%
\bar{x}	0.268	$x_1 \in [1 - \bar{x}, 1 + \bar{x}]$ Monthly separation rate = 4.2%
ψ	0.015	$\rho_z = .76, \sigma_z = 0.015$
\underline{z}	0.95	$\rho_z = .76, \sigma_z = 0.015$

Parameters used to match aggregate moments in the calibration of the model.

6 Results

The calibrated model matches the cross-sectional distribution of firms by size and age. Due to entry, exit, and productivity shocks, the simulated model produces varying employment growth rates within the distribution of firms.

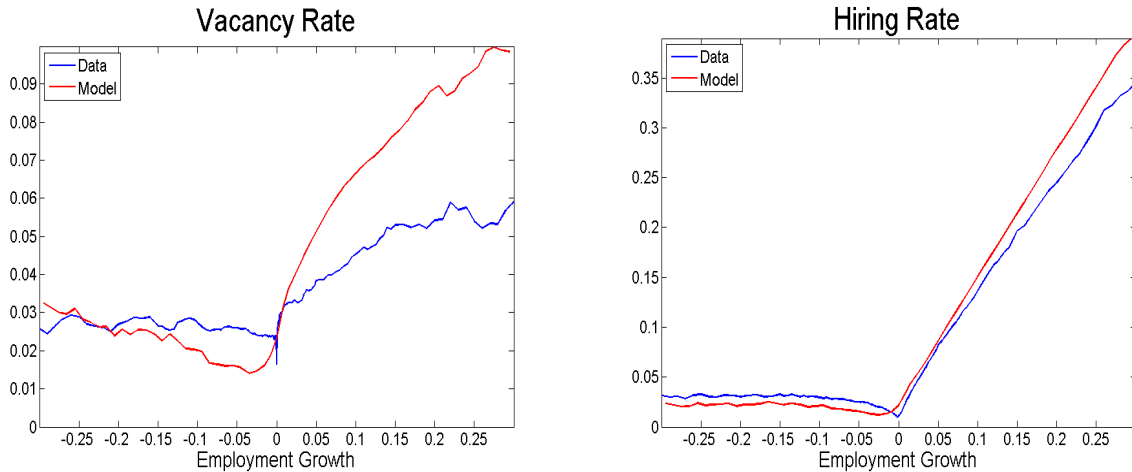
6.1 Recruitment, Part-time, and Separations by Employment Growth of Firms

One test of the model is in its ability to match the vacancy rates, hiring rate, layoff rate, and part-time rate used by firms over the employment growth distribution. Figure 12 shows that the model is able to match the patterns of vacancies and hires across the growth distribution of firms. Similar to Kaas and Kircher (2015), the model matches the broad pattern that vacancies are posted by growing firms, though the vacancy rate is higher than it is in the data. This can be matched more closely through a higher γ in the recruitment cost function. The data may also not entirely account for the increased recruitment of rapidly growing firms if multiple hires result from a single vacancy.

Figure 13 shows the growth rate of hours per worker, the layoff rate, and the part-time rate of firms by monthly employment growth rate. The model produces a positive correlation of layoff rates and part-time share of employment for contracting firms. This result is consistent with the evidence in CPS flow data that workers in part-time employment have high separation rates to unemployment. While layoffs are only used in contracting firms, some workers separate from all firms due to exogenous quits. Part-time usage, however, is exclusively used by shrinking firms. The likelihood of separating to unemployment given a worker has been placed into part-time work is therefore high, as some workers in the firm will also be laid off at the end of the period.

Although the monthly average part-time rate is lower than the layoff rate for shrinking firms, this is not necessarily contrary to the fact that full-time workers are more likely to

Figure 12: Vacancy Rates and Hiring Rates of Firms

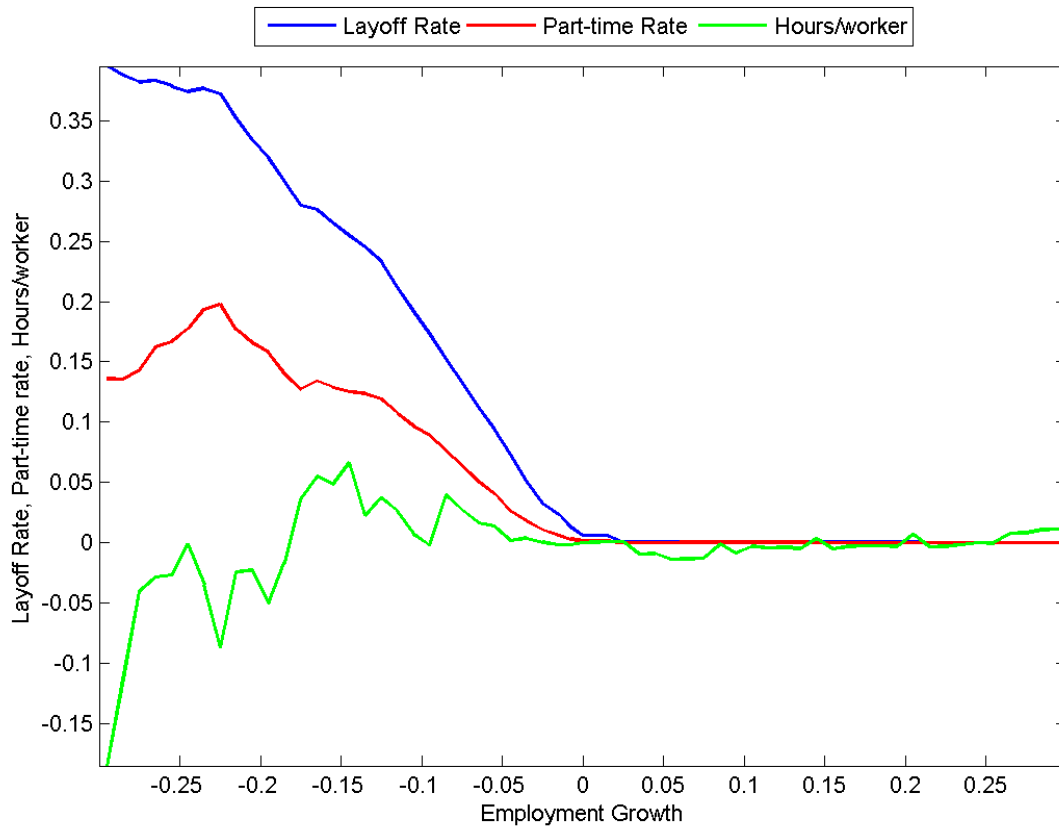


Monthly vacancy rates and hiring rates of firms, plotted by monthly employment growth rate. The blue lines are from data used in Davis et al. (2013). The red lines are model statistics.

move to PTE than to unemployment in the CPS. From the firm's individual policy function, Figure 9 in Section 4 showed that a contracting firm can use more part-time than separations in a period. While the layoff rate reflects total layoffs by a firm within the month, part-time percentage is an average of the weekly part-time utilization of a firm. The transitory nature of part-time usage and time-aggregation of total separations to a monthly rate can make the average part-time rate low in Figure 13, even if firms are moving more workers to part-time than using layoffs.

This short-term nature of part-time utilization in the model is also reflected in the growth-rate of weekly hours per worker for shrinking firms. While part-time usage and layoffs increase steadily for shrinking firms, the average monthly change in weekly hours per worker is relatively stable except at very negative growth-rates. This is due to fluctuations in part-time usage within a given month. If part-time usage were very persistent, it would be reflected in a steadier decline in the monthly growth rate of average weekly hours per worker.

Figure 13: Layoffs, Part-time, and Hours Growth



Layoffs, part-time, and weekly hours growth by monthly employment growth rate. The blue line is the layoff rate of firms, the red line indicates part-time utilization as a fraction of a firm's workforce, and the green line indicates the growth rate of average weekly hours for firms by employment growth.

6.2 Part-time Employment by Firm Age, Size, and Productivity

In addition to the distribution of part-time employment in firms by growth rate, the model produces interesting results for the cross-section of firms which use part-time labor by firm size and age category, as displayed in Table 6.2. It is interesting to note that the part-time share is not monotone in size categories, with the largest and smallest firms utilizing more part-time labor. This is due to the entry and growth of large firms, and reflects the fact that these size categories are somewhat arbitrary. For example, firms in the range of 249-999 employees in the model move across size categories with large enough shocks. Looking at the part-time share of firms by permanent productivity level, we can see that part-time usage is roughly monotone and increasing in size except at the highest productivity. The variation seen in part-time usage across productivity levels comes partly from the heterogeneous exit rates of firms, which affects the long-run size of a firm. If exit rates were constant across firms, average part-time usage would actually be decreasing in the firm's permanent productivity level. Instead, it is increasing, since more productive firms also have much lower exit rates. This raises the future value of the firm, giving the firm more incentive to retain its current workforce through part-time work. Fixing the productivity level, part-time usage is increasing in age and reflects the fact that young, growing firms use less part-time labor.

Table 6: Part-time Percentage by Firm Age and Size

Size class	1-49	50-249	250-999	1k-9,999	10k+
PT %	4.80	5.07	1.25	6.41	4.99
PT % \leq 2 years	3.34	4.08	1.03	4.06	3.21
PT % \leq 5 years	3.82	4.40	1.23	6.73	5.45
PT % \leq 10 years	3.99	4.83	1.28	6.81	3.95
Productivity (x_0)	$x_0 = 0.36$	$x_0 = 0.70$	$x_0 = 1.12$	$x_0 = 1.94$	$x_0 = 3.96$
PT %	3.71	4.71	5.28	6.41	4.99
PT % \leq 2 years	3.21	3.50	4.82	4.03	2.92
PT % \leq 5 years	3.56	4.33	5.11	6.71	5.32
PT % \leq 10 years	3.65	4.56	5.61	6.80	3.91

The top half of the table gives the average part-time rate of firms in each size class in total, and in firms of less than 2, 5, and 10 years of age. The bottom portion of the table gives these statistics for firms by their permanent productivity type, regardless of their current size.

Although the model can reproduce the qualitative properties of worker flow data and has testable implications for the usage of part-time employment over the distribution of firms, what is lacking is data on the patterns of hours adjustments or part-time usage of firms by age or size, both in the cross-section and over the business cycle. In future work, I plan to address this by documenting the hours and employment adjustments at the firm-level in the Longitudinal Business Database, Census of Manufactures/Annual Survey of Manufactures, and Quarterly Plant Capacity Utilization Survey data from the US Census Bureau.

6.3 Business Cycle Properties

I simulate the model with aggregate uncertainty to evaluate its ability to produce cyclical volatility in vacancies, unemployment, and part-time work over the business cycle. To evaluate the business cycle properties of the model, I focus on the relative volatility of part-time utilization, unemployment, vacancies, and worker flows in comparison to aggregate output. Two primary results arise from the business cycle analysis. First, the model is able to match the volatility and cyclical properties of *PTE* relative to *U* found in the data. That is, unemployment and part-time are countercyclical and similar in their relative volatility, as highlighted in Section 2.2. Second, the volatility of unemployment and vacancies is increased relative to the model without an operative part-time margin. Table 6.3 shows the business cycle moments for the model with and without part-time employment relative to the data. For the model without part-time labor, the model is recalibrated to the same target moments, but with $\ell = 0$. The primary change in parameter values between the two calibrations is the idiosyncratic shock parameters \bar{x} and π_x , which are .312 and 0.973 in the case without part-time (compared with $\bar{x} = .288$ and $\pi_x = .933$ with a part-time margin). The results displayed are the standard deviation and correlation of the cyclical component of the logged and detrended variables relative to aggregate output (*Y*).

The model with an operative part-time margin improves the fit of the model to business cycle properties substantially. Unemployment becomes more volatile and countercyclical,

Table 7: Business Cycle Statistics

	$\frac{sd(x)}{sd(Y)}$	$\rho(x, Y)$	$\frac{sd(x)}{sd(Y)}$	$\rho(x, Y)$	$\frac{sd(x)}{sd(Y)}$	$\rho(x, Y)$
	Data		Model		Model (No <i>PTE</i>)	
Prod/Worker	0.67	.89	0.92	.97	0.93	.97
U	7.47	-.85	4.96	-.46	2.88	-.38
PTE	6.50	-.83	3.86	-.98	NA	NA
Vacancies	6.81	.43	2.28	.56	1.20	.39
JF Rate	3.86	.81	2.73	.66	1.15	.48
Sep. Rate	2.67	-.58	2.80	-.15	2.66	-.23

though it is still not as volatile as it is in the data. The volatility of vacancies and the job-finding rate double. While the volatility in unemployment and vacancies increase, the separation rate remains close to the value in the data. Additionally, although the total volatility in *PTE* is higher in the data than in the model, the model matches quite well the relative volatility of *PTE* and *U*. The model also matches the strong negative correlation of *PTE* with aggregate output.

The change caused by part-time utilization on the business cycle properties in the model is twofold. First, the part-time margin changes the movement of measured labor productivity in reaction to a shock. A fundamental productivity shock causes changes along the part-time margin for firms, which stabilizes the change in labor productivity for the firm because of decreasing returns to scale in the production function. This decrease in utilized labor means that output is more volatile for a given shock. The increase in volatility of output is similar to the effect of varying effort in real business cycle models with labor-hoarding, such as in Burnside et al. (1993). Similar to their model, productivity shocks are realized after employment is fixed. In the labor-hoarding literature, however, hours per worker are fixed during a period since workers supply a constant number of hours, and firms adjust utilization by contracting with workers over effort when the shock is realized. This produces procyclical movements in labor productivity and increases the volatility of output from a shock. In this model, variation in labor utilization occurs in the hours margin through part-time employment rather than in unobservable effort. Firms adjust to productivity shocks

using part-time employment, changing the amount of output per worker. Productivity varies less than output since decreasing returns to scale increases the average productivity of firms when labor utilization decreases.

The second effect of part-time utilization is in the volatility of the model, and comes from the effect of part-time employment on the separation rate. Firms with the option of using part-time labor use fewer layoffs in the event of a large idiosyncratic productivity shock. Since the calibrated idiosyncratic shock process matches the average monthly separation rate, when firms use part-time instead of layoffs, a less persistent shock is needed to produce the same average monthly separation rate. This increases the volatility of unemployment and vacancies over the business cycle as well, since aggregate shocks primarily change the magnitude of job destruction and creation in adjusting firms. More firms adjusting labor produces a larger response to aggregate changes in productivity. One interesting point to note is that the volatility of the separation rate remains low as a result of the part-time margin being used instead of layoffs, so that the increase in volatility affects job creation more than job destruction.

It should be noted that this increased volatility arises even with a conservative value of leisure. The volatility of unemployment and vacancies can also be increased by raising the value of leisure b to be close to the value of working, as in the recalibration of the model of Mortensen and Pissarides (1994) by Hagedorn and Manovskii (2008). The reason volatility increases with an increase in b is that the surplus created by a filled vacancy decreases as workers become indifferent between leisure and work. This makes vacancies and the job-finding rate very sensitive to changes in productivity. However, raising the benefit from non-market work in this model will also result in a counterfactually high volatility of the separation rate.

7 Discussion

7.1 Ongoing Work with US Census Bureau Data

I show that the model can reproduce the qualitative properties of worker flow data at the firm level and over the business cycle. In addition, the model has implications for the cross-sectional distribution of part-time usage by firms. Firm growth causes young firms to use relatively little part-time labor, while the exit probability of firms changes the usage of part-time employment over the firm-size distribution. However, what is lacking along these dimensions is data on the patterns of hours adjustments or labor utilization of firms, both in the cross-section and over the business cycle. In ongoing work, I address this by documenting the cyclical and cross-sectional properties of hours and employment changes at the firm-level in US Census Bureau Data. I plan to document the patterns in employment growth and hours growth in manufacturing firms using the Longitudinal Business Database (LBD) and Census of Manufactures/Annual Survey of Manufactures (CMF/ASM). The goal is to document the correlation of hours and employment growth over the cross-sectional distribution of firms by size and age category. Also of interest is the cyclical volatility of these moments at the firm level by firm and establishment size, age and productivity. This is motivated by the findings of Moscarini and Postel-Vinay (2012), who document that the employment growth rate for large firms varies more over the business cycle than it does for small firms. Fort et al. (2013) show that young and small firms are very volatile over the cycle, while small but older firms have less cyclical employment growth. I plan to extend these facts to cover the response of hours growth rates to changes in aggregate unemployment and output by firm-level observable characteristics such as size and age. Understanding the properties of firm-level growth rates in hours and employment over the firm distribution will provide valuable insight into the dynamics of unemployment, employment, and labor utilization at the firm level.

7.2 Conclusion

In this paper, I document that fluctuations in part-time employment for economic reasons arise from within-job changes in hours due to slack work or business conditions. Motivated by this finding, I build a model to explain the cyclical movements in part-time employment based on firm-level changes in labor demand. I show that the model is capable of matching the patterns of worker transitions for part-time employment found in CPS data. Particularly, part-time employment utilization is transitory and characterized by high transitions to and from full-time employment. Firms also use part-time with layoffs, producing a high probability of separating to unemployment for part-time workers. Part-time utilization in the cross-sectional distribution of firms is dependent on firm characteristics such as age, size, and productivity. Over the business cycle, part-time employment and unemployment are countercyclical and display similar volatilities. Relative to the case with no part-time margin, the model can account for a significant increase in business cycle volatility in unemployment and vacancies.

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Appendix

A Appendix: Decentralized Problem

The firm of age j in history (x^j, z^t) takes as given its current stock of workers hired up to the current period, $(L_i)_{i=0}^{j-1}$, and the contracts signed by the previous labor cohorts, $(C_i)_{i=0}^{j-1}$. To not violate its prior commitments to past labor cohorts and be consistent with exogenous probabilities of separation and firm death, the firm's current exit probability must satisfy $\phi(\tau) \leq (1 - s_0)(1 - \delta)$ as well as $\delta \leq 1 - \phi(\tau)$, $\tau \leq j - 1$. This means that $s(\tau) = (1 - \phi(\tau))/(1 - \delta)$ when $\delta < 1$ (when $\delta = 1$, s can be arbitrary). The firm then decides the contract C_j to post in V vacancies, with C_j consistent with attracting desired matching probability m . When J_j is the value of a firm of age j , the firm's problem is:

$$J_j[(C_\tau)_{\tau=0}^{j-1}, (L_\tau)_{\tau=0}^{j-1}, x^j, z^t] = \max_{m, V, C_j} x_j z_t F(L(1 - \alpha_\tau(x^j, z^t)/2)) - \mathcal{W} - C(V, L) \\ + \beta(1 - \delta)\mathbb{E}_{x^j, z^t} J_{j+1}[(C_\tau)_{\tau=0}^j, (L_{\tau+})_{\tau=0}^j, x^{j+1}, z^{t+1}] \quad (22)$$

s.t.

$$L_{j+} = mV, \quad m \in [0, 1], \quad V \geq 0, \quad L_{\tau+} = L_\tau \frac{\phi_\tau(x^j, z^t)}{(1 - \delta)} \quad \forall \tau \leq j - 1 \quad (23)$$

$$s_0 \leq 1 - \phi_\tau(x^j, z^t)/(1 - \delta), \quad (24)$$

$$\mathcal{W} = \sum_{\tau=0}^{j-1} [w_{p,\tau}(x^j, z^t)\alpha_\tau(x^j, z^t) + w_{f,\tau}(x^j, z^t)(1 - \alpha_\tau(x^j, z^t))] L_\tau, \quad L = \sum_{\tau=0}^{j-1} L_\tau, \quad (25)$$

$$\rho(z^t) = \frac{m}{\lambda(m)}(1 - \delta)\beta\mathbb{E}_{x^j, z^t} [W(C_j, x^{j+1}, z^{t+1}) - U(z^{t+1})] \text{ if } m > 0 \quad (26)$$

The last condition is the workers' participation constraint, specifying the minimum utility a contract C_j must promise to attract a queue length of $\lambda(m)$ per vacancy.

A.1 Firm Entry

Free entry of firms implies that the expected value of an entrant firm (before realizing x and with labor force $L = 0$) is less than or equal to the entry cost K .

$$\sum_{x \in X} \pi(x_0) J_0[0, 0, x^0, z_t] \leq K(z_t) \quad (27)$$

If entry is positive then this condition holds with equality.

A.2 Competitive Equilibrium

A Competitive search equilibrium is a list

$$[U(z^t), W(\cdot), \rho(z^t), \mathcal{C}_j(x^j, z^t), V(x^j, z^t), J_j(\cdot), L_\tau(x^j, z^t), N(x^j, z^t), N_0(z^t)]$$

for all $t \geq 0$, $j \geq 0$, $x^j \in X^{j+1}$, $z^t \in Z^{t+1}$, $0 \leq \tau \leq j$ and given initial firm distribution, such that:

- Firms' exit, hiring, part-time, and layoff strategies are optimal. That is, J_a is the value function and $\mathcal{C}_j(\cdot)$, $\phi(\cdot) = (1 - \delta)(1 - s(\cdot))$, $\alpha(\cdot)$, $m(\cdot)$, and $V(\cdot)$ are the policy functions for problem (22) subject to its constraints (23)-(26).
- Employment evolves according to

$$L_\tau(x^j, z^t) = L_\tau(x^{j-1}, z^{t-1}) \frac{\phi(x^j, z^t)}{1 - \delta}, 0 \leq \tau \leq j - 1,$$

$$L_j(x^j, z^t) = m(x^j, z^t)V(x^j, z^t), j \geq 0$$

- Firm entry is optimal. That is, the complimentary slackness condition

$$\sum_x \pi_0(x) J_0(x, z^t) \leq K(z_t), N_0(z^t) \geq 0, \quad (28)$$

holds for all z^t , and the number of firms evolves according to their laws of motion (10) and (12).

- Workers' search strategies are optimal. That is, (ρ, U, E) satisfy equations (2), (3), and (1).
- Aggregate resource feasibility holds for all z^t :

$$\sum_{j \geq 0, x^j} N(x^j, z^t) \left[\lambda(m(x^j, z^t))V(x^j, z^t) + \sum_{\tau=0}^{j-1} L_\tau(x^j, z^t) \right] = 1. \quad (29)$$

A.3 Calibrated Firm Size and Age Distribution

Table A.3 provides the distribution of firm shares and employment share by size classification in the Business Dynamics Statistics data and in the calibrated model. The share of firms in each size category less than 2, 5, and 10 years old is also provided for the model and data.

B Appendix: Proofs

Proposition 3 *A Competitive search equilibrium is socially optimal.*

Size class	1-49	50-249	250-999	1k-9,999	10k+
Data					
Firm Shares	.956	.0364	.0054	.0017	.0002
Emp. Shares	.293	.162	.109	.176	.259
Share ≤ 2 years	.247	.072	.044	.023	.011
Share ≤ 5 years	.397	.169	.102	.054	.037
Share ≤ 10 years	.578	.313	.202	.120	.071
Model					
Firm Shares	.956	.0365	.006	.0014	.0002
Emp. Shares	.329	.166	.119	.152	.236
Share ≤ 2 years	.161	.029	.022	0.02	.015
Share ≤ 5 years	.355	.083	.068	.062	.046
Share ≤ 10 years	.583	.169	.137	.128	.098

The proof proceeds in two steps: First, the participation constraint is substituted into the firm's problem, and using the workers' recursive equations, we show that the firms' objective function is identical to the planner's objective function for the maximization of the social surplus of the firm when $\rho(z^t) = \mu(z^t)$. Second, we show that the choice sets of the firm and planner coincide. We can then show that the firm's entry decision coincides with optimal entry of firms when $\rho(z^t) = \mu(z^t)$. We then show that the competitive equilibrium allocation satisfies the conditions for Lemma B.1. Thus, the competitive search equilibrium is socially optimal since it is a solution to the sequence problem of the planner.

Proof Define the social surplus of the firm of age j with history (x^j, z^t) , and predetermined employment and contracts as

$$\mathcal{G}_j[(\mathcal{C}_\tau)_{\tau=0}^{j-1}, (L_\tau)_{\tau=0}^{j-1}, x^j, z^t] \equiv J_j[(\mathcal{C}_\tau)_{\tau=0}^{j-1}, (L_\tau)_{\tau=0}^{j-1}, x^j, z^t] + \sum_{\tau=0}^{j-1} L_\tau [W(\mathcal{C}_\tau, x^j, z^t) - U(z^t)] \quad (30)$$

Using (1) and (3), the worker surplus satisfies:

$$\begin{aligned} W(\mathcal{C}_\tau, x^j, z^t) - U(z^t) &= [w_{f,j}(x^j, z^t)(1 - a_\tau(x^j, z^t)) + w_{p,\tau}(x^j, z^t)a_\tau(x^j, z^t)] \\ &\quad - (1 - \ell a_\tau(x^j, z^t))b - \rho(z^t) \\ &\quad + \beta \phi_\tau(x^j, z^t) \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})] \end{aligned}$$

Substituting this equation and (22) into (30) and using $\sigma \equiv (\mathcal{C}_\tau, x^j, z^t)$ and $\sigma_+ \equiv$

$(\mathcal{C}_\tau, x^{j+1}, z^{t+1})$ along with $L_{\tau+} = L_\tau \phi_\tau / (1 - \delta) \forall \tau \leq j - 1$ and $L = \sum_{\tau=0}^{j-1} L_\tau$ to get:

$$\begin{aligned}
\mathcal{G}_j(\sigma) &= \max_{\delta, m, V, \mathcal{C}_j} \left\{ x_j z_t F(L(1 - a_\tau(x^j, z^t)/2)) - C(V, L(1 - a_\tau(x^j, z^t)/2), x z_t) - f \right. \\
&\quad - \sum_{\tau=0}^{j-1} L_\tau \left[[w_{f,\tau}(x^j, z^t)(1 - a_\tau(x^j, z^t)) + w_{p,\tau}(x^j, z^t)a_\tau(x^j, z^t)] \right. \\
&\quad \left. \left. + \beta(1 - \delta)\mathbb{E}_{x^j, z^t} J_{j+1}(\sigma_+) \right] \right\} \\
&\quad + \sum_{\tau=0}^{j-1} L_\tau \left[[w_{f,\tau}(x^j, z^t)(1 - a_\tau(x^j, z^t)) + w_{p,\tau}(x^j, z^t)a_\tau(x^j, z^t)] \right. \\
&\quad \left. - (1 - \ell a_\tau(x^j, z^t))b - \rho(z^t) + \beta \phi_\tau(x^j, z^t)\mathbb{E}_{x^j, z^t} [W(\mathcal{C}_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})] \right] \quad (31)
\end{aligned}$$

$$\begin{aligned}
&= \max_{\delta, m, V, \mathcal{C}_j} \left\{ x^j z^t F(L(1 - a_\tau(x^j, z^t)/2)) \right. \\
&\quad - \left[\rho(z^t) + (1 - \ell a_\tau(x^j, z^t))b \right] L \\
&\quad - C(V, L(1 - a_\tau(x^j, z^t)/2), x z_t) - f + \beta(1 - \delta)\mathbb{E}_{x^j, z^t} J_{j+1}(\sigma_+) \\
&\quad \left. + \beta \sum_{\tau=0}^{j-1} L_\tau \phi_\tau(x^j, z^t)\mathbb{E}_{x^j, z^t} [W(\mathcal{C}_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})] \right\} \quad (32)
\end{aligned}$$

$$\begin{aligned}
&= \max_{\delta, m, V, \mathcal{C}_j} \left\{ x_j z_t F(L(1 - a_\tau(x^j, z^t)/2)) - [(1 - \ell a_\tau(x^j, z^t))b] L \right. \\
&\quad - \rho(z^t)[L + \lambda(m)V] - f - C(V, L(1 - a_\tau(x^j, z^t)/2), x z_t) + \beta(1 - \delta)\mathbb{E}_{x^j, z^t} J_{j+1}(\sigma_+) \\
&\quad \left. + \beta(1 - \delta) \sum_{\tau=0}^j L_{\tau+} \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})] \right\} \quad (33)
\end{aligned}$$

$$\begin{aligned}
&= \max_{\delta, m, V, \mathcal{C}_j} \left\{ x_j z_t F(L(1 - a_\tau(x^j, z^t)/2)) - [(1 - \ell a_\tau(x^j, z^t))b] L \right. \\
&\quad \left. - \rho(z^t)[L + \lambda(m)V] - f - C(V, L(1 - a_\tau(x^j, z^t)/2), x z_t) + \beta(1 - \delta)\mathbb{E}_{x^j, z^t} G_{j+1}(\sigma_+) \right\} \quad (34)
\end{aligned}$$

Where (32) makes use of the fact that the wage bill of the firm and the total wages of all cohorts are equal, and the sums of search value and leisure utility for all cohorts can be aggregated to be expressed in terms of L .

Step (33) makes use of the fact that $L_{\tau+} = L_{\tau}\phi_{\tau}/(1 - \delta)$ to get the last line of (32) to be

$$\begin{aligned}
& \beta \sum_{\tau=0}^{j-1} L_{\tau}\phi_{\tau}(x^j, z^t) \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_{\tau}, x^{j+1}, z^{t+1}) - U(z^{t+1})] \\
&= \beta(1 - \delta) \sum_{\tau=0}^{j-1} L_{\tau+} \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_{\tau}, x^{j+1}, z^{t+1}) - U(z^{t+1})] \\
&= \beta(1 - \delta) L_{j+} \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_{\tau}, x^{j+1}, z^{t+1}) - U(z^{t+1})] \\
&+ \beta(1 - \delta) \sum_{\tau=0}^{j-1} L_{\tau+} \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_{\tau}, x^{j+1}, z^{t+1}) - U(z^{t+1})] \tag{35}
\end{aligned}$$

Then, using the participation constraint (26) in the firm's problem in the LHS of (35), we can write

$$\rho(z^t) \frac{\lambda(m)}{m} L_{j+} = \rho(z^t) \lambda(m) V = \beta(1 - \delta) \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_j, x^{j+1}, z^{t+1}) - U(z^{t+1})]$$

to get to (34).

This shows that the objective function maximized by the firm subject to (26) coincides with the objective function of the planner for maximizing the social surplus of the firm, (34), when $\rho(z^t) = \mu(z^t)$. Since the firm and the planner have the same objective function, it remains to show that the firm's choice set with cohort-specific contracts is equivalent to the planner's choice set using identical policies for all workers. That is, each feasible allocation for the planner using identical policies for all workers is attainable by a combination of cohort-specific contracts, and vice versa. Since there is no time-inconsistency problem, it is sufficient to show that the choice sets of both sequence problems are equivalent at the time of firm entry.

First, I show that any allocation feasible for the planner to achieve for a given firm at entry is also feasible for the firm. Consider the sequential formulation of the planner's problem as specified in (9), and let a feasible allocation for the planner for a firm at entry be a list of sequences $(\mathbf{L}, \mathbf{V}, \mathbf{m}, \mathbf{s}, \mathbf{a}, \delta)$ with $\mathbf{L} = (L(x^j, z^t))_{j,t \geq 0}$ and similar notation for the other variables, where $m(x^j, z^t) \in [0, 1]$, $V(x^j, z^t) > 0$, $s(x^j, z^t) \in [s_0, 1]$, $a(x^j, z^t) \in [0, 1]$, $\forall x^j, z^t$ and \mathbf{L} and \mathbf{N} evolve according to their laws of motion for all $j \geq 0, t \geq 0$. Note that the planner's policies are identical for the entire stock of workers for a given firm. Let the firm's allocation be described by a list of sequences $((\mathbf{L}_{\tau})_{\tau=0}^{j-1} (\mathcal{C}_{\tau})_{\tau=0}^{j-1}, \mathbf{V}, \mathcal{C}_j, \delta)$, with $\mathbf{L}_{\tau} = (L_{\tau}(x^j, z^t))_{j,t \geq 0}$ and similar notation for contracts and exit probabilities. Note that the firm chooses a history-dependent and cohort-specific sequence of contracts. To show that the firm can reproduce any allocation feasible for the planner, we can set the vector of firm's policies \mathbf{V} , and δ identical to the planner for each history, and set for each cohort τ and history (x^j, z^t) , $s_{\tau}(x^j, z^t) = s(x^j, z^t)$, $a_{\tau}(x^j, z^t) = a(x^j, z^t) \forall j, \tau \geq 0$. The value of new contract \mathcal{C}_j is set to attract the queue length that produces the same matching probability as the planner, $m(x^j, z^t)$ in each history. Then for each cohort τ , L_{τ} evolves so that $L = \sum_{\tau=0}^{j-1} L_{\tau}$ for the firm's total labor for all cohorts in each history, which is identical to the planner's stock of labor. Note that the firm can mimic the planner's allocation because it has no need to

commit to separation rates or part-time employment rates in future histories to reach any value of contract \mathcal{C}_j . This is because the firm can use wages $w_{p,\tau}(x^j, z^t)$ and $w_{f,\tau}(x^j, z^t)$ in any history to alter the value of contract \mathcal{C}_j to attract the appropriate queue length without committing to a or s that would be inconsistent with the planner's choice in any history.

To show that any feasible allocation for the firm to achieve at entry is also feasible for the planner, we must show that the planner can achieve in any history an identical total labor force, hiring and exit policies, and total separation and part-time labor rate as the firm achieves through cohort-specific policies. To do this, we simply can set the planner's choice of $m(x^j, z^t)$ and $V(x^j, z^t)$ to match the vacancies $V(x^j, z^t)$ chosen by the firm in all histories, as well as the matching rate m chosen implicitly through the value of contract $\mathcal{C}_j(x^j, z^t)$. Similarly, the planner can set $\delta(x^j, z^t)$ to be identical to the firm's choice in every history. Lastly, if the planner's policy satisfies $s(x^j, z^t)L(x^j, z^t) = \sum_{\tau=0}^{j-1} s_\tau(x^j, z^t)L_\tau(x^j, z^t)$ and $a(x^j, z^t)L(x^j, z^t) = \sum_{\tau=0}^{j-1} a_\tau(x^j, z^t)L_\tau(x^j, z^t)$ at each history (x^j, z^t) , then the labor stock of the planner will be identical to the sum of labor stocks over all $j - 1$ cohorts of the firm. Since any allocation of labor in each history can be replicated by the planner, the choice set of the planner and the firm are identical. Since we showed they also maximize the same objective function when $\rho(z^t) = \mu(z^t)$ in (31), the maximization problem of the entrant firm and the planner's social value of the firm at entry are identical.

It remains to show that firm entry is also socially efficient when $\rho(z^t) = \mu(z^t)$. Since the social value of an entrant firm coincides with the firm's value function at entry, we can show that this value coincides with the planner's value of an entering firm, $G_0(0, x, z^t)$ as specified in (36) when $\rho(z^t) = \mu(z^t)$. So the free entry condition in competitive equilibrium (28) coincides with the condition for socially optimal firm entry (15). Because of aggregate resource feasibility in equilibrium (29), the planner's resource constraint (13) is also satisfied. Since the allocation of a competitive search equilibrium satisfies all the conditions of B.1, it is a solution of the planning problem (9) and is thus socially optimal.

Proposition 4 • *Suppose that a solution of (14) and (15) exists with associated allocation $\mathbf{A} = (\mathbf{N}, \mathbf{L}, \mathbf{V}, \mathbf{m}, \mathbf{s}, \mathbf{a}, \delta)$ satisfying $N(z^t) > 0$ for all z^t . Then \mathbf{A} is a solution to the sequential planning problem (9).*

- *If $K(z)$, f , and b are sufficiently small and if $z_1 = \dots = z_n = \bar{z}$, equations (14) and (15) have a unique solution (G, M) . If the transition matrix $\psi(z_j|z_i)$ is strictly diagonally dominant and if $|z_i - \bar{z}|$ is sufficiently small for all i , equations (14) and (15) have a unique solution.*

Proof Let $\beta^t \psi(z^t) \mu(z^t) \geq 0$ be the multiplier on the resource constraint (13) in history z^t . Then $\mu(z^t)$ is the social value of a worker in history z^t . For the vector of multipliers $\mu = (\mu(z^t))$, Let $G_t(L, x, z^t)$ denote the value of an existing firm with employment L and productivity x in aggregate productivity history z^t . The sequence G_t obeys the recursive equations

$$G_t(L, x, z^t) = \max_{\delta, s, a, V, m} x z_t F(L(1 - a/2)) - (1 - \ell a)bL - f - \mu(z^t)[L + \lambda(m)V] - C(V, L(1 - a/2), x z_t) + \beta(1 - \delta)\mathbb{E}_{x, z^t} G(L_+, x_+, z^{t+1}) \quad (36)$$

s.t.

$$L_+ = (1 - s)L + mV, \delta \in [\delta_0, 1], a \in [0, 1], s \in [s_0, 1], m \in [0, 1] \text{ and } V \geq 0.$$

It first must be shown that (36) is equivalent to the planner's problem (9) via Lemma (B.1). Then it must be shown that the reduced problem (14) solves (36) if entry is strictly positive in all states.

Lemma B.1 • *For given multipliers $\mu(z^t)$, there exist value functions $G_t : \mathbb{R}_+ \times X \times Z^{t+1} \rightarrow \mathbb{R}$, $t \geq 0$, satisfying the system of recursive equations (36).*

- *If $\mathbf{X} = (\mathbf{N}, \mathbf{L}, \mathbf{V}, \mathbf{m}, \mathbf{s}, \mathbf{a}, \delta)$ is a solution of the planning problem (9) with multipliers $\mu = (\mu(z^t))$, then the corresponding firm policies also solve problem (36) and the complementary-slackness condition*

$$\sum_{x \in X} \pi_0(x) G_t(0, x, z^t) \leq K(z_t), N_0(z^t) \geq 0 \quad (37)$$

is satisfied for all z^t .

Conversely, if \mathbf{X} solves for every firm problem (36) with multipliers μ , and if conditions (37) and the resource constraint (13) hold for all z^t , then \mathbf{X} is a solution of the planning problem (9).

Proof Part a): Same as in Kaas and Kircher (2015). Show the operator T has a unique fixed point which is a sequence of bounded functions.

Part b): Taking a solution \mathbf{X} of the planning problem and writing $\beta^t \psi^t \mu(z^t) \geq 0$ for the multipliers on constraints (13), then the solution \mathbf{X} maximizes the Lagrangian:

$$\begin{aligned} \mathcal{L} = \max_{t \geq 0, z^t} \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) N_0(z^t) + \sum_{j \geq 0, x^j} N(x^j, z^t) \left[x_j z_t F((1 - a(x^j, z^t))/2) L(x^j, z^t) \right. \right. \\ \left. \left. - b(1 - \ell a(x^j, z^t)) L(x^j, z^t) - C(V(x^j, z^t), (1 - a(x^j, z^t))/2) L(x^j, z^t), x_j z_t) \right. \right. \\ \left. \left. - \mu(z^t) [L(x^j, z^t) + \lambda(m(x^j, z^t)) V(x^j, z^t)] \right] \right\} \quad (38) \end{aligned}$$

Which is the sequential formulation of the recursive problem (36) with multipliers $\mu(z^t)$. This implies that the firm policies from the planner's problem yielding allocation \mathbf{X} also solve the recursive problem. The Lagrange function can be re-arranged to be the sum of the social values of entrant firms and the social values of existing firms of idiosyncratic history x^j at $t = 0$ (and aggregate history z^0).

$$\begin{aligned} \mathcal{L} = \max_{N_0(\cdot)} \sum_{t, z^t} \beta^t \psi(z^t) N_0(z^t) \left[-K(z^t) + \sum_x \pi_0(x) G_t(0, x, z^t) \right] \\ + \sum_{z \in Z} \psi(z^0) \sum_{j \geq 1, x^j} N(x^j, z^t) G_0(L(x^j, z^0), x_j, z_0) \quad (39) \end{aligned}$$

This rearrangement makes clear that condition (37) describes optimal entry of firms. Since the solution to the planner's problem yields an allocation \mathbf{X} which is also the maximum of the sequential sum of recursive firm-level problems for all histories, the solution to (9) is also a solution to (36) with (37) satisfied for all aggregate histories z^t .

To prove the converse, suppose \mathbf{X} solves for every firm the recursive problem (36) with given multipliers $\mu(z^t)$, and that (37) and the resource constraint (13) are satisfied. The proof that \mathbf{X} solves the original sequential planning problem (9) subject to (13) follows by contradiction: Suppose there is an alternate allocation $\hat{\mathbf{X}}$ for the sequential planning problem under constraint (13) that strictly dominates \mathbf{X} . Denoting the net output created by firm (x^j, z^t) in \mathbf{X} as

$$O(x^j, z^t) \equiv x_j z_t F((1 - a(x^j, z^t)/2)L(x^j, z^t)) - b(1 - \ell a(x^j, z^t))L(x^j, z^t) - C(V(x^j, z^t), (1 - a(x^j, z^t)/2)L(x^j, z^t), x_j z_t)$$

and $\hat{O}(x^j, z^t)$ for $\hat{\mathbf{X}}$. Then total surplus \hat{S} in allocation $\hat{\mathbf{X}}$ satisfies

$$\begin{aligned} \hat{S} &= \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) \hat{N}_0(z^t) + \sum_{j \geq 0, x^j} \hat{N}(x^j, z^t) \hat{O}(x^j, z^t) \right\} \\ &= \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) \hat{N}_0(z^t) + \mu(z^t) - \mu(z^t) + \sum_{j \geq 0, x^j} \hat{N}(x^j, z^t) \hat{O}(x^j, z^t) \right\} \\ &\leq \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) \hat{N}_0(z^t) + \mu(z^t) \right. \\ &+ \left. \sum_{j \geq 0, x^j} \hat{N}(x^j, z^t) [\hat{O}(x^j, z^t) - \mu(z^t) (\hat{L}(x^j, z^t) + \lambda(\hat{m}(x^j, z^t)) \hat{V}(x^j, z^t))] \right\} \\ &\leq \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \hat{N}_0(z^t) \left[-K(z_t) + \sum_x \pi_0(x) G_t(0, x, z^t) \right] \\ &+ \sum_{z \in Z} \psi(z^0) \sum_{j \geq 0, x^j} N(x^j, z^0) G_0(L(x^j, z^0), x_j, z^0) + \sum_{t, z^t} \beta^t \psi(z^t) \mu(z^t) \\ &\leq \sum_{t \geq 0, z^t} \beta^t \psi(z^t) N_0(z^t) \left[-K(z_t) + \sum_x \pi_0(x) G_t(0, x, z^t) \right] \\ &+ \sum_{z \in Z} \psi(z^0) \sum_{j \geq 0, x^j} N(x^j, z^0) G_0(L(x^j, z^0), x_j, z^0) + \sum_{t, z^t} \beta^t \psi(z^t) \mu(z^t) = S. \end{aligned}$$

The first equality follows from adding and subtracting μ . The first inequality comes from the resource constraint (13). The second inequality follows from the discounted sum of surplus values for an individual firm of age j at t being

$$\begin{aligned} \sum_{\tau \geq t} \beta^{\tau-t} \sum_{x^{j+\tau-t}, z^\tau} \psi(z^\tau | z^t) \pi(x^{j+\tau-t} | x^j) \prod_{k=t}^{\tau-1} [1 - \delta(x^{j+k-t}, z^k)] [O'(x^{j+\tau-t}, z^\tau) \\ - \mu(z^\tau) [\hat{L}(x^{j+\tau-t}, z^\tau) + \lambda(\hat{m}(x^{j+\tau-t}, z^\tau)) \hat{V}(x^{j+\tau-t}, z^\tau)]] \end{aligned}$$

being bounded above by $G_t(0, x_0, z^t)$ (for new firms, $a = 0$) or by $G_0(L(x^j, z^0), x^j, 0)$ (for firms of age j existing at $t = 0$) by the definition of G_t . The third inequality follows from the complementary slackness condition (37). Either $(-K(z_t) + \sum_x \pi_0(x)G_t(0, x, z^t))$ is zero, in which case the first sum is zero on both sides of the inequality, or the term is negative, in which case $N_0(z^t) = 0$ and $N'_0(z^t) \geq 0$, making the final expression weakly larger. The equality follows from the definition of the surplus value S and the original assumption that allocation \mathbf{X} solves (36).

This proves $S' \leq S$ and hence contradicts the hypothesis that $S' > S$. This completes the proof of Lemma B.1.

C Appendix: Data

C.1 Constructing the CPS sample and flow data

I extract the following variables from the CPS raw data, downloaded from NBER.

The variables needed to reproduce the gross flows analysis is listed in Table 8, including the variable name and description.¹⁶ Consecutive months of the CPS are merged by matching household identifier, state, and person number. Additional checks are used to ensure an actual match such as age, race, and gender.¹⁷ Once the merged files are created, the labor market transition of each matched individual is created from *PEMLR* and *PRWKSTAT*. Flows are then constructed and weights applied (*fweight*) to produce monthly flow probabilities for each possible labor market state, differentiating employed individuals between full-time employment, part-time employment for noneconomic reasons, and part-time employment for economic reasons. The weighted flows are then seasonally adjusted. I do not make any correction for time-aggregation as is used in Shimer (2005) or Elsby et al. (2009).

The main variables used to classify workers by labor force and part-time or full-time status are *PEMLR*, *PRFTLF*, *PRPTREA*, and *PRWKSTAT*. The variable *PEMLR* gives the individual’s labor force status, whether employed, unemployed, or not in the labor force. *PRFTLF* gives the individual’s part-time or full-time labor force status by whether they usually work 1 – 34 or 35+ hours per week, regardless of current employment or hours actually worked in the survey week. *PRPTREA* gives the reason that any worker classified as full-time who worked 1-34 hours or any worker classified as part-time or usually part-time (*PEHRUSLT* = 0-34 OR *PEHRACTT* = 1-34) gives for why they are working part-time, along with their usual part-time or full-time status. Those that report “Slack Work/Business Conditions”, “Could Only Find PT Work”, and “Seasonal Work” are considered part-time (or usually part-time) for economic reasons. Those who choose any other reason for working part-time (such as “Child Care Problems” or “Health/Medical Limitations”) are classified as part-time (or usually part-time) for non-economic reasons. *PRWKSTAT* then classifies individuals as full-time or part-time work status based on actual hours worked, usual part-

¹⁶The variable name in the STATA code is in parentheses, if changed.

¹⁷For dates where consecutive months cannot be linked, the flows are linearly interpolated. This occurs in Jun-Sept. 1995, Sept-Oct. 2009, and Sept.-Oct. 2009. The September dates are where Labor Day corresponds with the reference week in the survey, causing a counterfactual spike in the number individuals reporting “at work part-time for noneconomic reasons.”

Table 8: CPS Variables Used In Producing Labor Flows

HRHHID	1-12	Household identifier
HRMIS	63-64	Month-in-sample (mis)
HRLONGLK	69-70	Longitudinal link indicator (llind)
HRSEUSUF	75-76	Serial suffix
GESTFIPS	93-94	FIPS State code
PEAGE	122-123	Person's age as of the end of survey week (age)
PESEX	129-130	Sex (sex)
PTDTRACE	139-140	Race (race)
PULINENO	147-148	Person's line number (line)
PEMLR	180-181	Monthly labor force recode (status)
PEHRUSLT	224-226	How many hours a week do you usu. work at your job/jobs?
PEHRACTT	247-249	Last week, how many hrs did you actually work at your job/jobs?
PRPTREA	405-406	Detailed reason for part-time
PRWKSTAT	416-417	Full/part-time work status
PWSSWGT	613-622	Final weight (fweight)

time or full-time status, and economic or non-economic reasons. Those who list economic reasons for part-time work but report not being available for or wanting full-time work are also classified as part-time for non-economic reasons. The actual BLS definition I use to separate employed workers is the classification of “At work” by full- or part-time and by reason. This corresponds to those who actually worked 35 or more or less than 35 hours the past week. If 1-34 hours was worked, those who gave economic reasons and reported availability and desire to work full-time are classified as “At work part-time for economic reasons”, and those who are not able or willing to work full-time or give non-economic reasons for working less than 35 hours in the reference week are classified as “At work part-time for noneconomic reasons.” Note that this classification leaves out absent workers who count as employed, and that part- vs. full- time is determined by actual hours worked, not usual part- or full-time status. The values for variable *PRWKSTAT* are listed in Table 9. Following BLS definitions in Statistics (1997), I classify $PRWKSTAT = 2, 8, 9$ as “at work full-time”, $PRWKSTAT = 3, 6$ as “at work part-time for economic reasons”, and $PRWKSTAT = 4, 7$ as “at work part-time for non-economic reasons”.¹⁸ Absent workers are not included in “At work” full-time or part-time.¹⁹

C.2 PTE workers by Origin

Figure 14 plots the shares of the current stock of *PTE* workers by their status the prior month. The majority of workers reported as at work part-time for economic reasons come from employment (the red and green lines). Only about 10% of *PTE* workers flowed from

¹⁸The following results are also robust to classification using classification by usual hours worked, ($FT:PRWKSTAT = 2$, $PTE:PRWKSTAT = 3, 6, 8$ and $PTN:PRWKSTAT = 4, 7, 9$)

¹⁹For stocks, the monthly, seasonally adjusted series for total employment, part-time for economic reasons, and part-time for non-economic reasons for nonagricultural industries are pulled from FRED (Series LNS12035019, LNS12032197, and LNS12032200).

Table 9: Values for PRWKSTAT

-1	Not in Universe
1	Not in Labor Force
2	FT Hours (35+), Usually FT
3	PT for Economic Reasons, Usually FT
4	PT for Non-economic Reasons, Usually FT
5	Not at Work, Usually FT
6	PT Hours, Usually PT for Economic Reasons
7	PT Hours, Usually PT for Non-economic Reasons
8	FT Hours, Usually PT for Economic Reasons
9	FT Hours, Usually PT for Non-economic Reasons
10	Not at Work, Usually PT
11	Unemployed FT
12	Unemployed PT

U or N in a given month. It is clear that this flow of workers from nonemployment is also constant over the business cycle.

C.3 Cyclical characteristics of all flows

Table 10: Cyclical Properties of PTE outflows

From	<i>PTE</i>	<i>PTE</i>	<i>PTE</i>	<i>PTE</i>	<i>PTE</i>
To	<i>FT</i>	<i>PTN</i>	<i>PTE</i>	<i>U</i>	<i>N</i>
$std(x)$.077	.073	.097	.092	.105
$std(x)/std(Y)$	3.25	3.11	4.12	3.91	4.45
$corrcoef(Y, x)$.855	.693	-.873	-.255	.458

Table 11: Cyclical Properties of U outflows

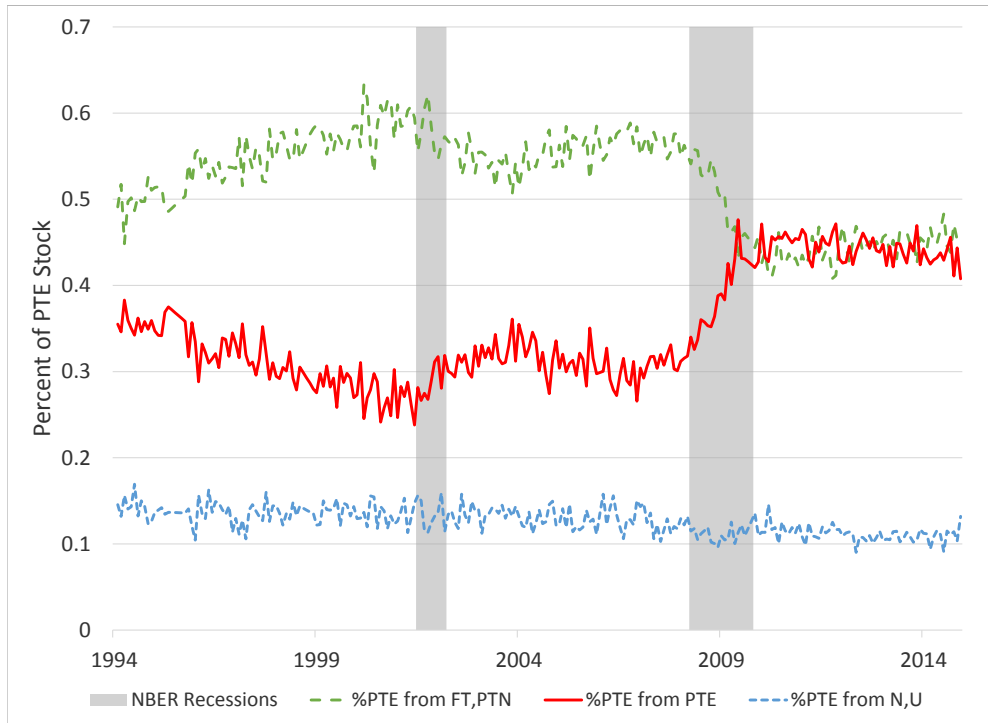
From	<i>U</i>	<i>U</i>	<i>U</i>	<i>U</i>	<i>U</i>
To	<i>FT</i>	<i>PTN</i>	<i>PTE</i>	<i>U</i>	<i>N</i>
$std(x)$.171	.185	.075	.080	.068
$std(x)/std(Y)$	7.26	7.88	3.18	3.41	2.90
$corrcoef(Y, x)$.808	.711	-.245	-.718	0.579

Table 12 gives the standard deviation and relative volatility to that of aggregate output, and correlation with aggregate output of the cyclical component of the logged and H-P filtered stocks and flow probabilities for all labor market flows.

Table 12: Cyclical Properties of Labor Market Statistics

	Y	FT	PTN	PTE	u rate
$std(x)$.024	.027	.024	.174	.190
$std(x)/std(Y)$	1	1.15	1.00	7.39	8.08
$corrcoef(Y, x)$	1	.906	.826	-.861	-.795
	$FT - FT$	$FT - PTN$	$FT - PTE$	$FT - U$	$FT - N$
$std(x)$.009	.075	.150	.110	.070
$std(x)/std(Y)$	0.39	3.19	6.39	4.68	2.96
$corrcoef(Y, x)$.380	-.135	-.773	-.525	.598
	$PTN - FT$	$PTN - PTN$	$PTN - PTE$	$PTN - U$	$PTN - N$
$std(x)$.052	.022	.147	.068	.055
$std(x)/std(Y)$	2.19	0.96	6.24	2.89	2.36
$corrcoef(Y, x)$.137	.218	-.830	-.597	.512
	$PTE - FT$	$PTE - PTN$	$PTE - PTE$	$PTE - U$	$PTE - N$
$std(x)$.077	.073	.097	.092	.105
$std(x)/std(Y)$	3.25	3.11	4.12	3.91	4.45
$corrcoef(Y, x)$.855	.693	-.873	-.255	.458
	$U - FT$	$U - PTN$	$U - PTE$	$U - U$	$U - N$
$std(x)$.171	.185	.075	.080	.068
$std(x)/std(Y)$	7.26	7.88	3.18	3.41	2.90
$corrcoef(Y, x)$.808	.711	-.245	-.718	0.579
	$N - FT$	$N - PTN$	$N - PTE$	$N - U$	$N - N$
$std(x)$.102	.063	.140	.125	.003
$std(x)/std(Y)$	4.34	2.66	5.93	5.30	0.11
$corrcoef(Y, x)$.786	.764	-.604	-.777	.258

Figure 14: Shares of PTE stock by Origin



Share of *PTE* in a given month by reported labor force status the prior month. The green line indicates workers in *PTE* who reported being in *FT* and *PTN* the prior month. The red line indicates the share of *PTE* workers who reported being *PTE* the prior month. The blue line indicates workers who reported being unemployed or nonparticipants the prior month.

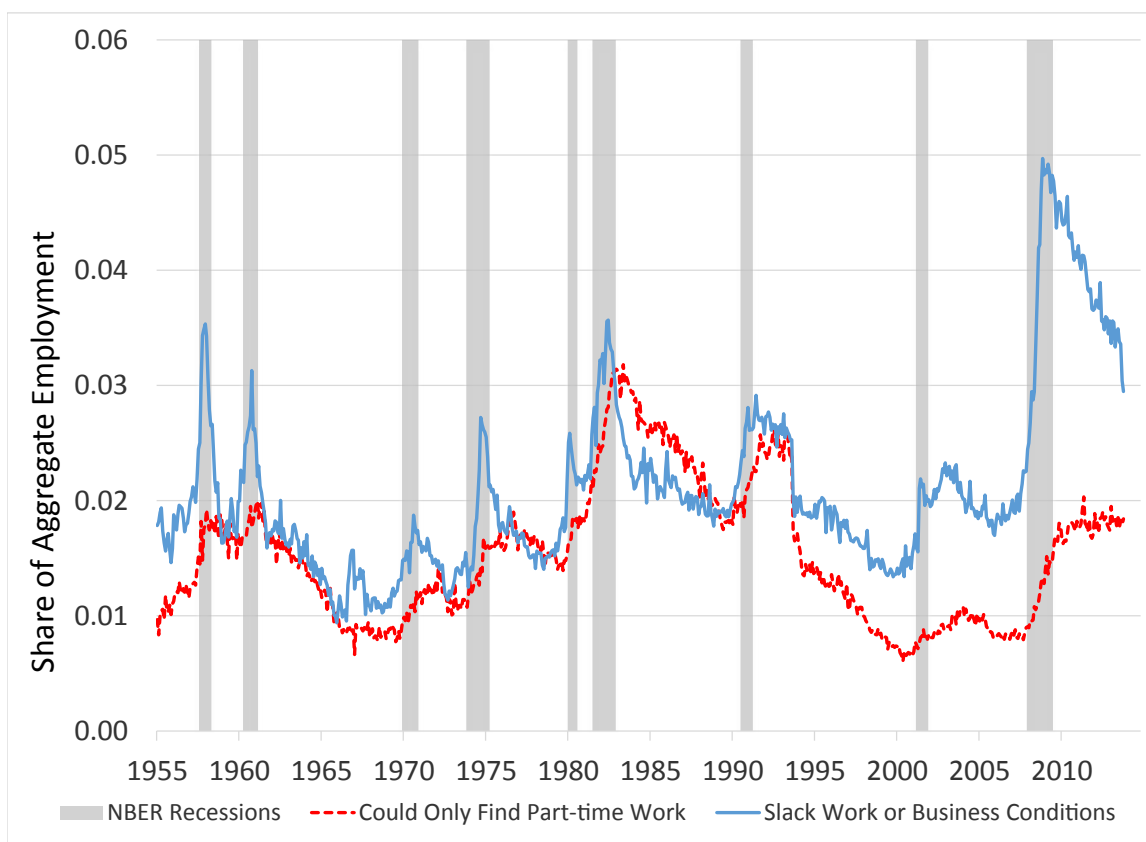
C.4 Distinguishing Slack Work and Failed Full-time Searchers

The reasons reported by workers as to why they are employed part-time that are classified as “economic reasons” are “Slack Work/Business Conditions,” “Failed to Find a Full-time Job,” and “Seasonal Work.” One concern about lumping these reasons into one population is that the behavior and flows for workers by detailed reason may be very different, and hence be important to distinguish. For instance, one might reasonably expect many failed full-time searchers to have come from unemployment rather than full-time employment the previous month. Similarly, a part-time employed worker due to slack work/business conditions might be more likely to have come from previously being employed full-time rather than having come from nonemployment. It could also be the case that the types of jobs held by failed full-time searchers is different than those who are in *PTE* due to slack work/business conditions. If the jobs held by the slack work group were once full-time positions with their current employer, it could be that the worker is more likely to gain full-time status from increased hours demanded in the same job, while a failed full-time searcher may only reach full-time

status by switching jobs. In this section, I look at the stocks and flows of part-time and full-time employment, but further disaggregated into part-time by detailed reason. I focus on the two subsets of “Slack Work” and “Failed Full-time Search” for PTE . I make the argument that the two groups behave similarly enough that the distinction between SW and FFT is not very large, especially relative to the differences between PTN and PTE .

The stocks are again taken from FRED, while flow probabilities are constructed from the basic monthly CPS files from 1994-2014. To produce the labor market states, the variables PEMLR and PRWKSTAT are used to categorize FT, PTN, U , and N . If a worker is classified as PTE , PRPTREA is used to create one population of slack work SW and one group of failed full-time searchers FFT .

Figure 15: Part-time Employment for Economic Reasons: Share of Aggregate Employment



The main difference in the stocks of SW and FFT are their relative size and cyclical-ity. Looking at stocks before the redesign, the two groups are very similar in size. After the redesign, SW seems to increase substantially as the reason for part-time employment, especially during the Great Recession. Figure 15 plots the shares of employment of both groups. What is noticeable in the graphs is the sharper increases and decreases in the slack work group during recessions. When looking at flows, however, the distinction between the two groups is not so clear.

From the average monthly flow probabilities, we can see that the flows of part-time employed due to slack work and the flows of failed full-time searchers are similar. The main differences in the outflows involves the flow probability from each group to full-time

Table 13: US Average Monthly Flow Probabilities: CPS 1994:1-2014:12

From	To					
	FT	PTN	SW	FFT	U	N
FT	.829	.077	.011	.002	.009	.014
PTN	.278	.531	.025	.013	.017	.061
SW	.333	.199	.248	.050	.053	.038
FFT	.190	.220	.112	.296	.065	.046
U	.115	.067	.020	.019	.492	.225
N	.015	.021	.001	.001	.025	.901

employment. The persistence and separation rates to nonemployment are very similar. The second major difference between the two groups is the flow probability of full-time and voluntary part-time workers into *SW* is larger than the flow probability from these groups to *FFT*. This reflects the idea that slack work is largely comprised of workers who did not switch employers, but rather had their hours with their current employer cut below 35 hours per week.

Although this may seem like evidence that these two populations are distinct, if we look at the workers who flow into the failed full-time search category in a given month, only about 18% of these workers transitioned from nonemployment. On average, 50% of workers in *FFT* came from *FT*, *PTN*, or *SW* the previous month. For workers who were part of the *SW* inflow, about 10% came from nonemployment and 64% came from *FT*, *PTN*, or *FFT*. The flow probabilities of workers from unemployment and nonparticipation to either *PTE* group is also nearly identical.

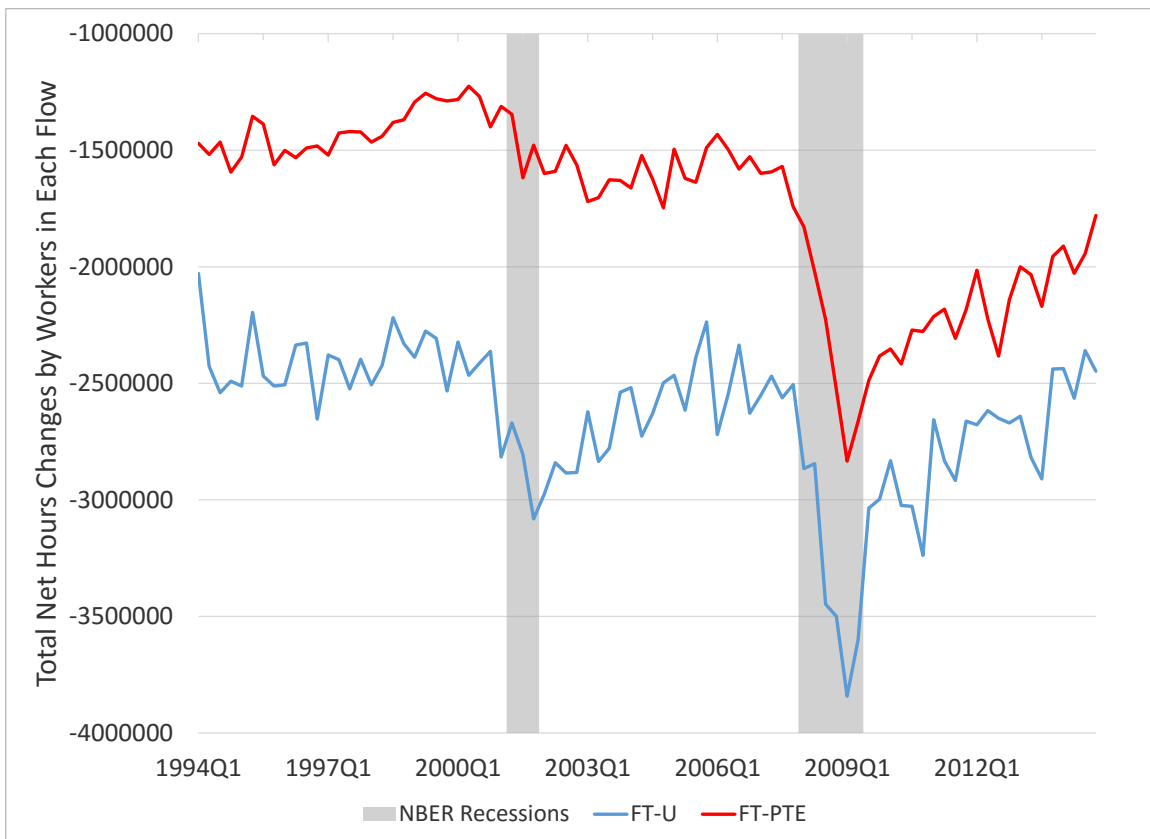
Another cause for concern in making a distinction between *PTE* workers by detailed reason is that the flow probabilities between all three part-time categories is quite large. It is very likely that there is significant misclassification in these part-time categories, given the high flow probabilities between groups. One possibility is that workers reporting failed full-time search is more an indicator of whether they are actively engaged in on-the-job-search rather than the origin of the part-time employed worker or the nature of the job. The idea that the *FFT* group consists of workers who came from nonemployment and ended up settling for a part-time job is not entirely supported by the flows data. One final concern is that further disaggregating *PTE* by detailed reason adds additional noise to the CPS flow probabilities due to a very small sample size for some flows, especially to and from *FFT*. Due to the lack of clear differences in the inflow or outflow probabilities of these *PTE* groups by detailed reason, I focus only on the categories of part-time employment by economic vs. non-economic reasons in this analysis.

C.5 Aggregate Hours Fluctuations and Part-time for Economic Reasons

The business cycle fluctuations in aggregate hours worked are similar in magnitude to real output per capita, while employment is roughly two-thirds as volatile over the business

cycle. Hours per worker is only about one-third as volatile as per capita output.²⁰ Of interest is the extent to which these cyclical fluctuations in hours can be accounted for by the transition between full-time and part-time labor, or if hours fluctuations are largely within these categories. The total of aggregate hours accounted for by workers in *PTE* does fluctuate cyclically as expected, as seen in Figure 17. However, the net change in aggregate hours due to flows in and out of *PTE* is a function of the size of these flows and their originating stocks. The sum of hours changes experienced by workers flowing into and out of *PTE* are substantial and cyclical, but offsetting. Figure 16 shows that the total of hours changes experienced by workers who transition from *FT* to *PTE* is nearly as large as the total loss in hours from workers who move from *FT* to *U*.

Figure 16: Sum of Net Hours Loss by Workers in Each Flow



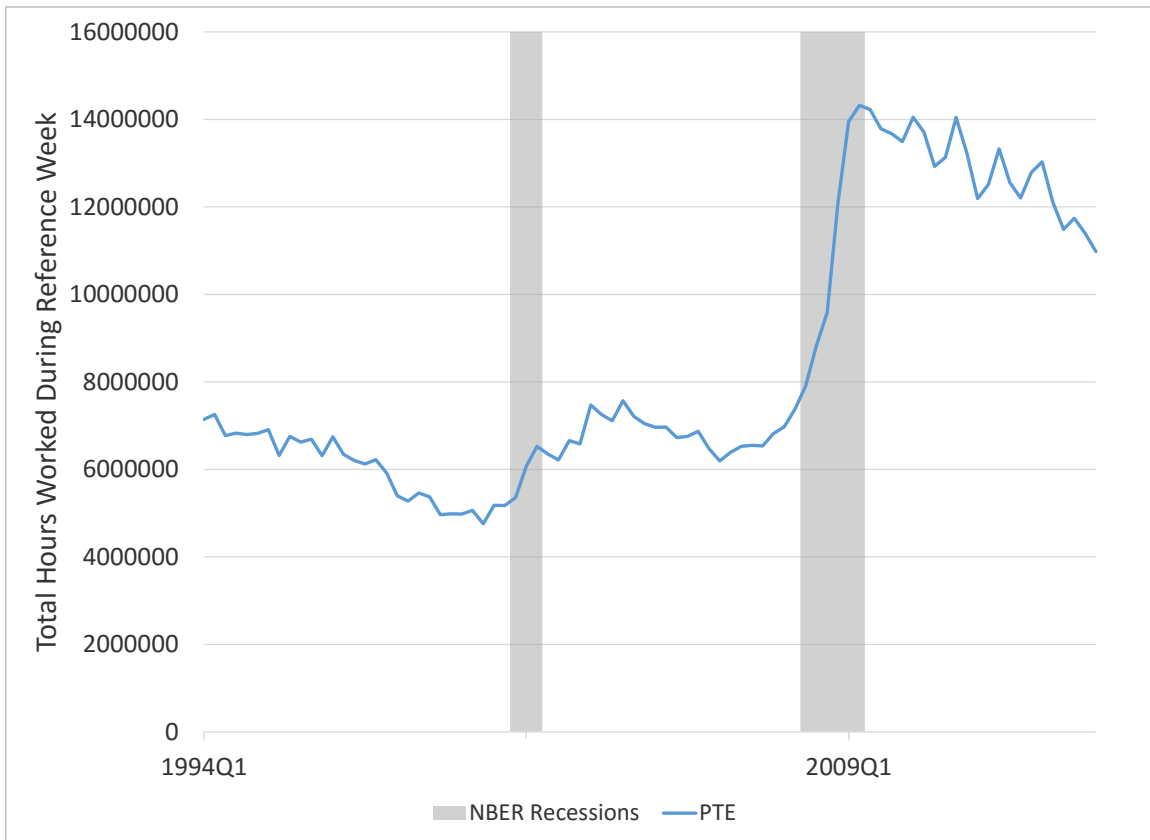
Each solid line plots the quarterly averaged total of the month-to-month change in reported weekly hours worked by all workers who experienced a transition from full-time employment to either *PTE* or *U*. The dotted line plots the total hours lost by all workers who either transitioned from *FT* to *PTE* or separated from *PTE* to *U*.

Since aggregate hours worked is a stock whose changes are governed by both hours changes and the size of each flow, it is important to note that the change in hours due to a flow may be cyclical even if the flow probability itself is constant. If the stock of unemployment increases, the hours loss from the *PTE* – *U* flow increases along with the

²⁰See Andolfatto (1996)

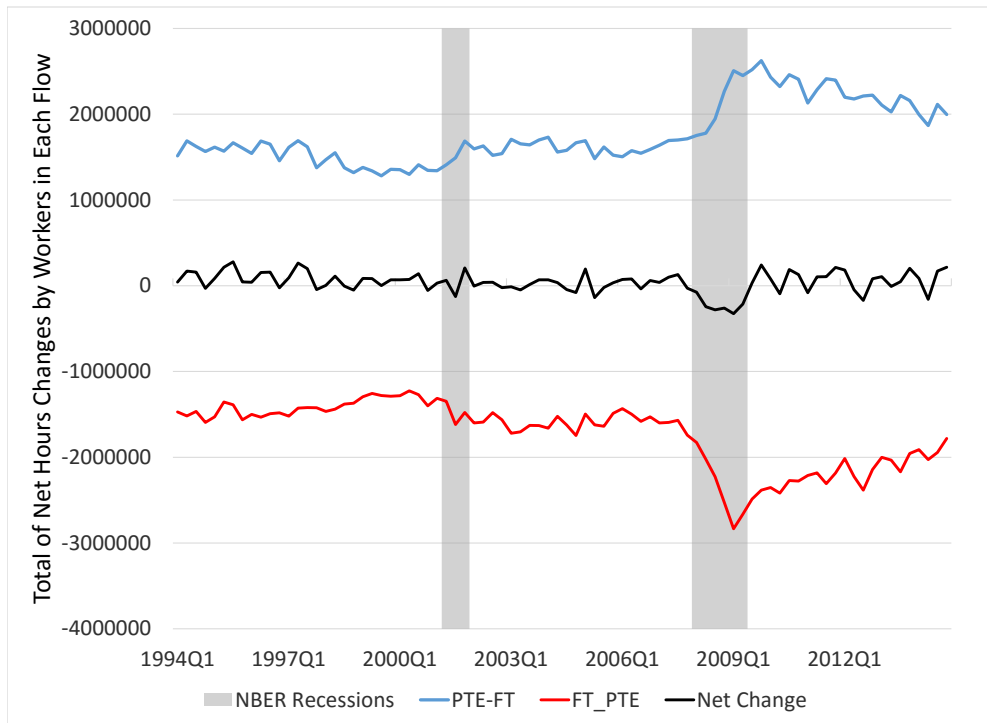
number of workers moving to *PTE* from *U* due to the larger pool of unemployed. Thus, the fluctuation in hours may be cyclical even if the flow probability from *PTE* to *U* is nearly constant. Similarly, cyclical flow probabilities may produce only a small contribution to aggregate hours fluctuations if their stocks or their hours contributions are offsetting. For example, while the *FT* – *PTE* flow and *PTE* – *FT* flow are negatively correlated, The countercyclical *FT* – *PTE* flow reduces hours and the procyclical *PTE* – *FT* flow increases hours. If stocks were held constant, both flows should work to make aggregate hours more countercyclical. Figure 18 shows that the hours changes contributed by workers in each flow offset to produce a much smaller net change. Since the *PTE* stock grows substantially in recessions, the total number of workers (and hours gained) moving to *FT* increases even though the transition probability decreases. This offsets much of the hours loss due to the increase in the *FT* – *PTE* flow. Figure 19 shows the relative magnitude of the gain in hours from the flow of workers to *FT*. Interestingly, the most cyclical flow probability (*UE*) is the least cyclical in terms of its contribution to the change in aggregate hours.

Figure 17: Total hours worked by individuals classified in *PTE*



Quarterly averaged total of the weekly hours worked by all workers who report being employed part-time for economic reasons.

Figure 18: Sum of Net Hours Gain/Loss by Workers in $PTE - FT$ and $FT - PTE$ flows

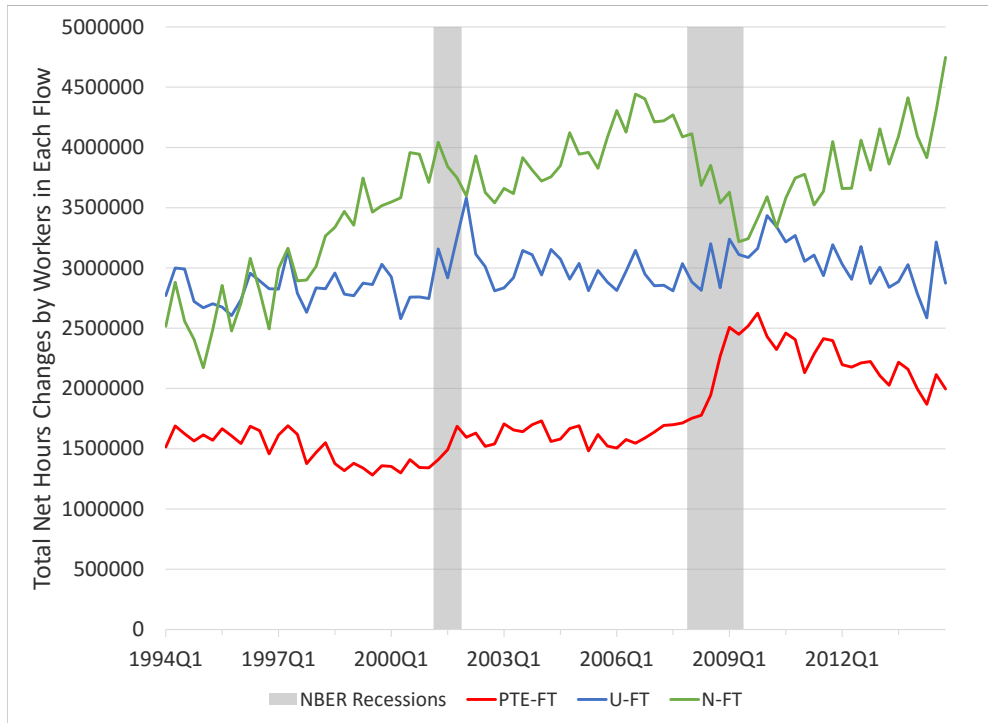


Each solid line plots the quarterly averaged total of the month-to-month change in reported weekly hours worked by all workers who experienced a transition between PTE and FT .

C.6 The Survey of Income and Program Participants

The Survey of Income and Program Participants consists of multiple panels of household surveys over varying lengths of time. In this study, I focus on the 1996 panel of the SIPP. This panel surveys about 60,000 individuals every four months for 48. The survey asks individuals retroactively their labor force participation/employment, work hours, and earnings at monthly frequency during each survey period. Due to the retroactive nature of the data, the SIPP responses to labor market questions differ slightly from their counterparts in the CPS. In CPS surveys, individuals are asked about labor market behavior for a specific reference week. Responses to questions on part-time work capture only those who report less than 35 hours of work in the reference week. In SIPP surveys, the respondent is asked whether they worked less than 35 hours in any week during the month. Respondents in the SIPP who report having worked part-time are not asked whether they were available for or desired full-time work at the time. They are asked to provide a reason for working part-time, including “Slack work or material shortage” and “Could not find a full-time job.” Similar to the CPS before the redesign, I classify as PTE any respondent who reported working

Figure 19: Sum of Net Hours Loss by Workers in Each Flow



Each line plots the quarterly averaged total of the month-to-month change in reported weekly hours worked by all workers who experienced a transition to full-time employment from either *PTE*, *U*, or *N*.

part-time during the reference month and provided either of the afore-mentioned reasons for working part-time. Respondents who report any other reason for part-time work are classified as *PTN*.